BAYESIAN BIG BANG

Fred Daum

Information Universe Conference
Groningen

7 October 2015

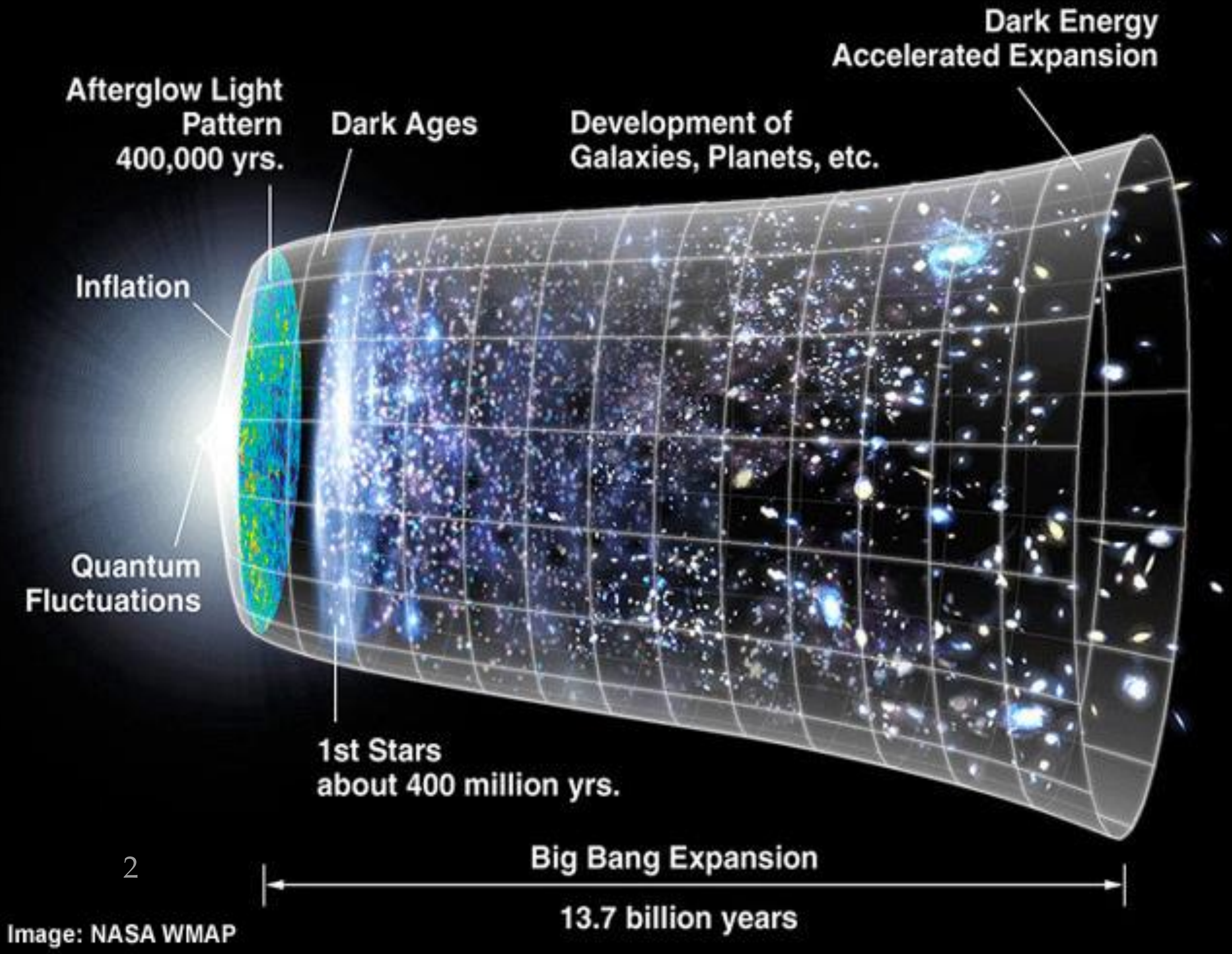
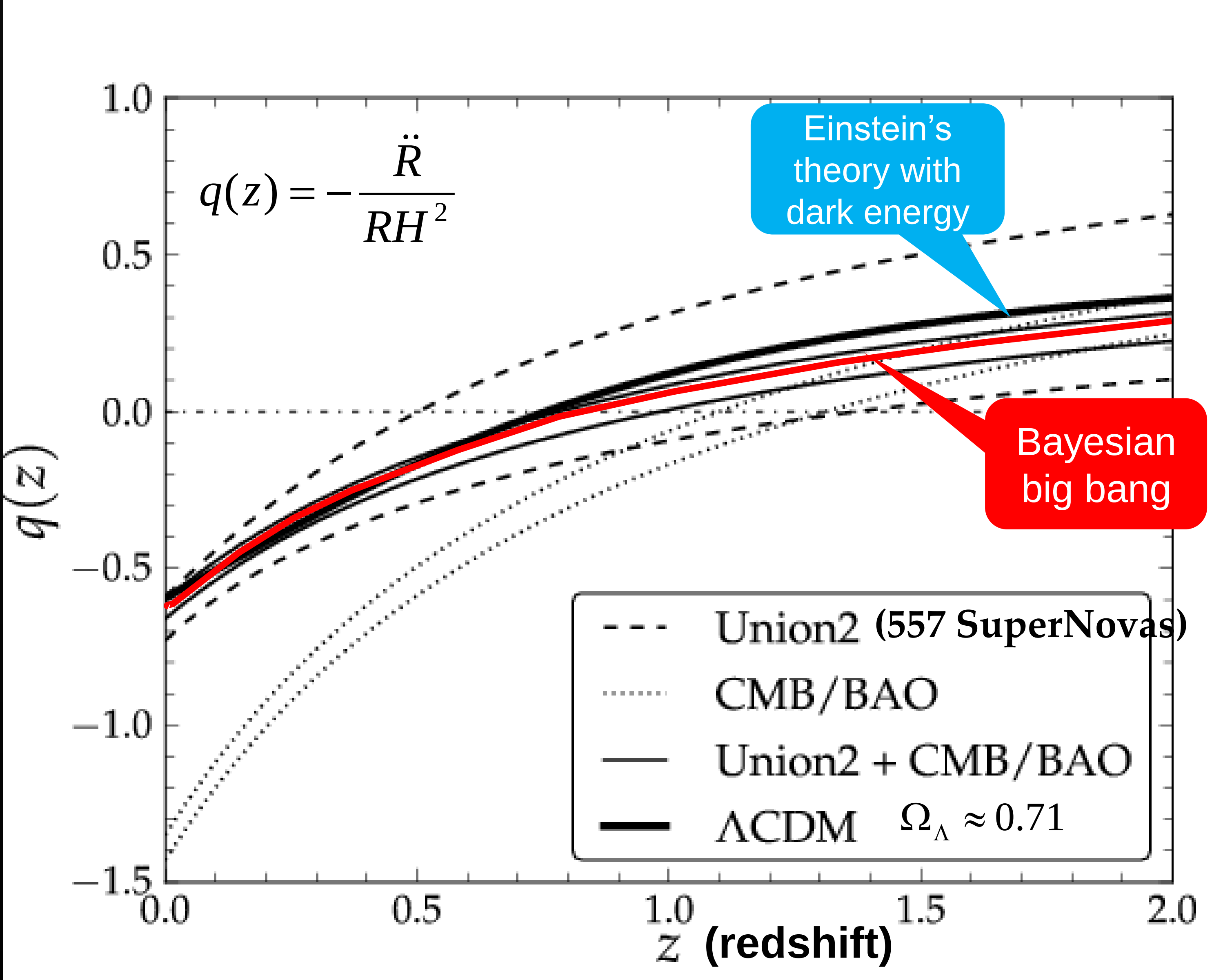


Image: NASA WMAP



$$q(z) = -\frac{\ddot{R}}{RH^2}$$

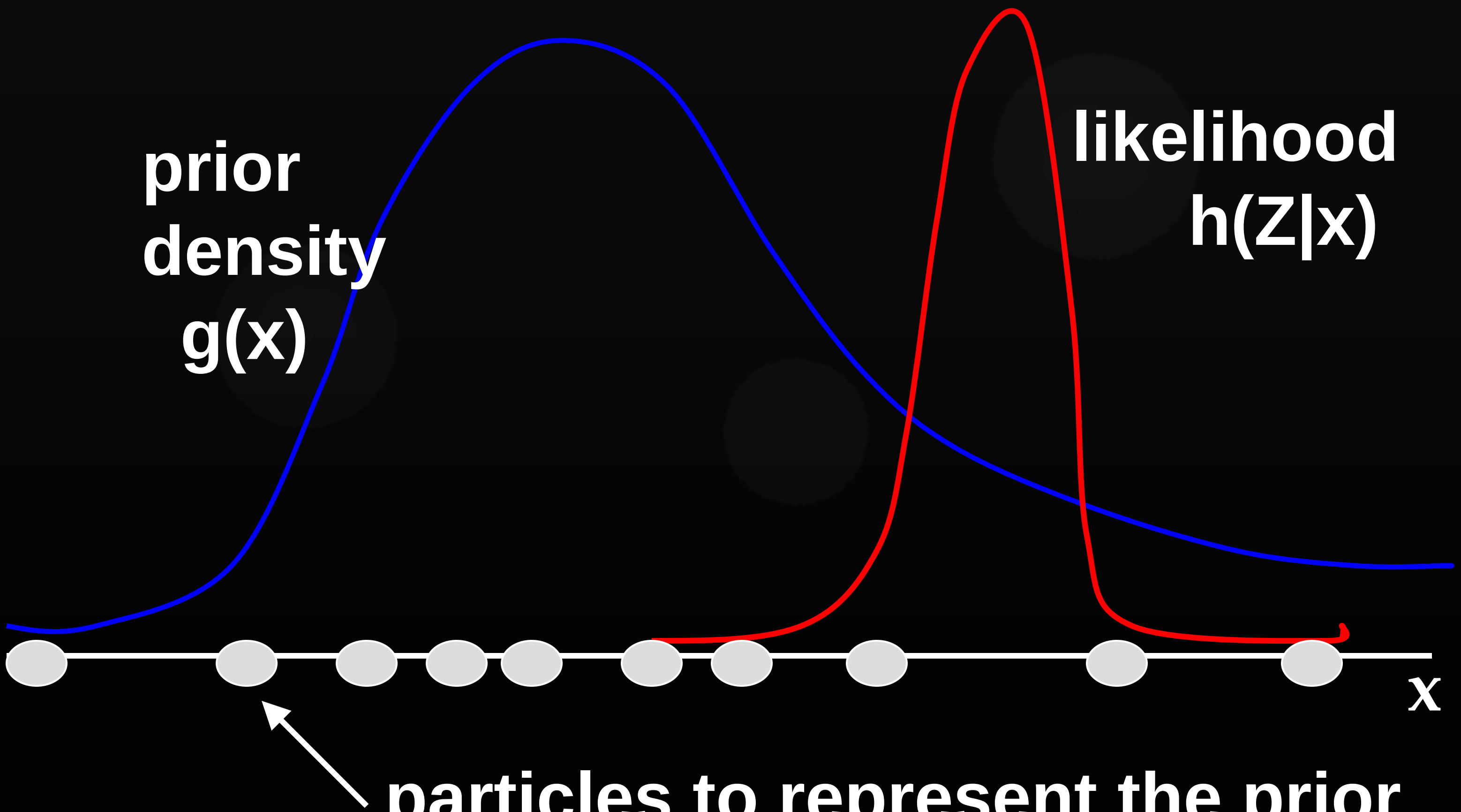
$\Omega_\Lambda \approx 0.71$

Einstein's theory with dark energy

Bayesian big bang

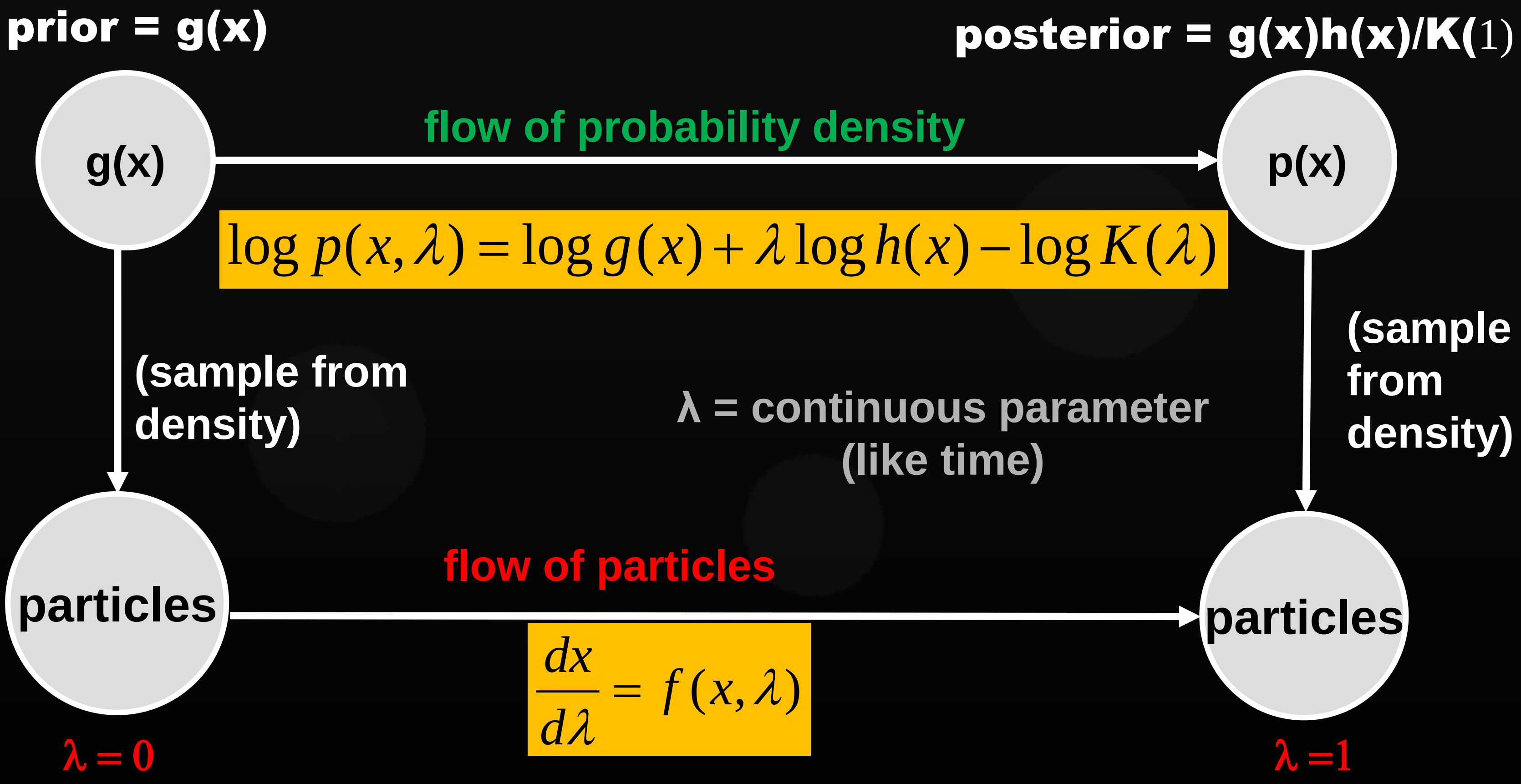
B. Santos, et al. (Nov 2013) arXiv:1009.2733 [astro-ph.CO]

BAYES' RULE: $p(x|Z) = g(x)h(Z|x)/K$



particles to represent the prior

induced flow of particles for Bayes' rule



prior = g(x)

posterior = g(x)h(x)/K(1)

g(x)

flow of probability density

p(x)

$$\log p(x,\lambda) = \log g(x) + \lambda \log h(x) - \log K(\lambda)$$

(sample from density)

λ = continuous parameter (like time)

(sample from density)

particles

flow of particles

particles

$$\frac{dx}{d\lambda} = f(x,\lambda)$$

$\lambda = 0$

$\lambda = 1$

$$div(pf) = p\left[-\log h + \frac{d \log K}{d\lambda}\right]$$

$$div(pf) = \left[\frac{d \log K(\lambda)}{d\lambda} - \log h(x) \right] p(x,\lambda)$$

$$\frac{dx}{d\lambda} = f(x,\lambda) = \left[\frac{\partial V(x,\lambda)}{\partial x} \right]^T / p(x,\lambda)$$

$$Tr\left[\frac{\partial^2 V(x,\lambda)}{\partial x^2} \right] = \left[\frac{d \log K(\lambda)}{d\lambda} - \log h(x) \right] p(x,\lambda)$$

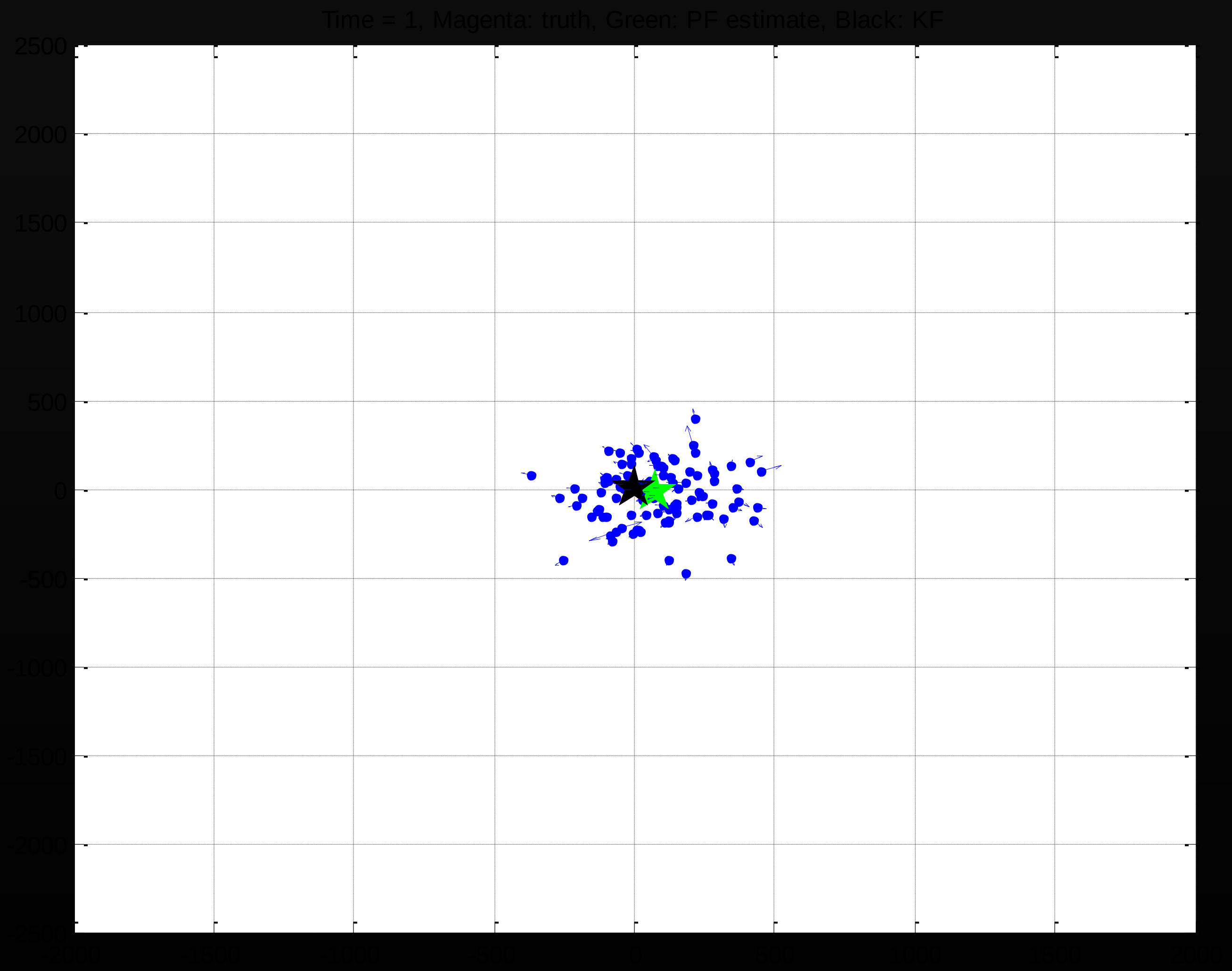
assuming that $\mathrm{d} \geq 3$ and $\mathrm{p}(\mathrm{x},\lambda) \rightarrow 0$ sufficiently fast as $\|\mathrm{x}\| \rightarrow 0$ we get:

$$V(x,\lambda) = \int \left[\log h(y) - \frac{d \log K(\lambda)}{d\lambda} \right] p(y,\lambda) \frac{c}{\|x-y\|^{d-2}} dy$$

$$\frac{\partial \mathrm{V}(\mathrm{x},\lambda)}{\partial \mathrm{x}} = \int \left[\log h(y) - \frac{d \log K(\lambda)}{d\lambda} \right] p(y,\lambda) \frac{c(2-d)(x-y)^T}{\|x-y\|^d} dy$$

$$\frac{\partial V(x,\lambda)}{\partial x} = E\left[\left(\log h(y) - \frac{d \log K(\lambda)}{d\lambda} \right) c(2-d)(x-y)^T / \|x-y\|^d \right]$$

$$\frac{\partial V(x_i,\lambda)}{\partial x} \approx \frac{1}{N} \sum_{j=1}^{N} \left(\log h(x_j) - \frac{d \log K(\lambda)}{d\lambda} \right) \frac{c(2-d)(x_i - x_j)^T}{\|x_i - x_j\|^d}$$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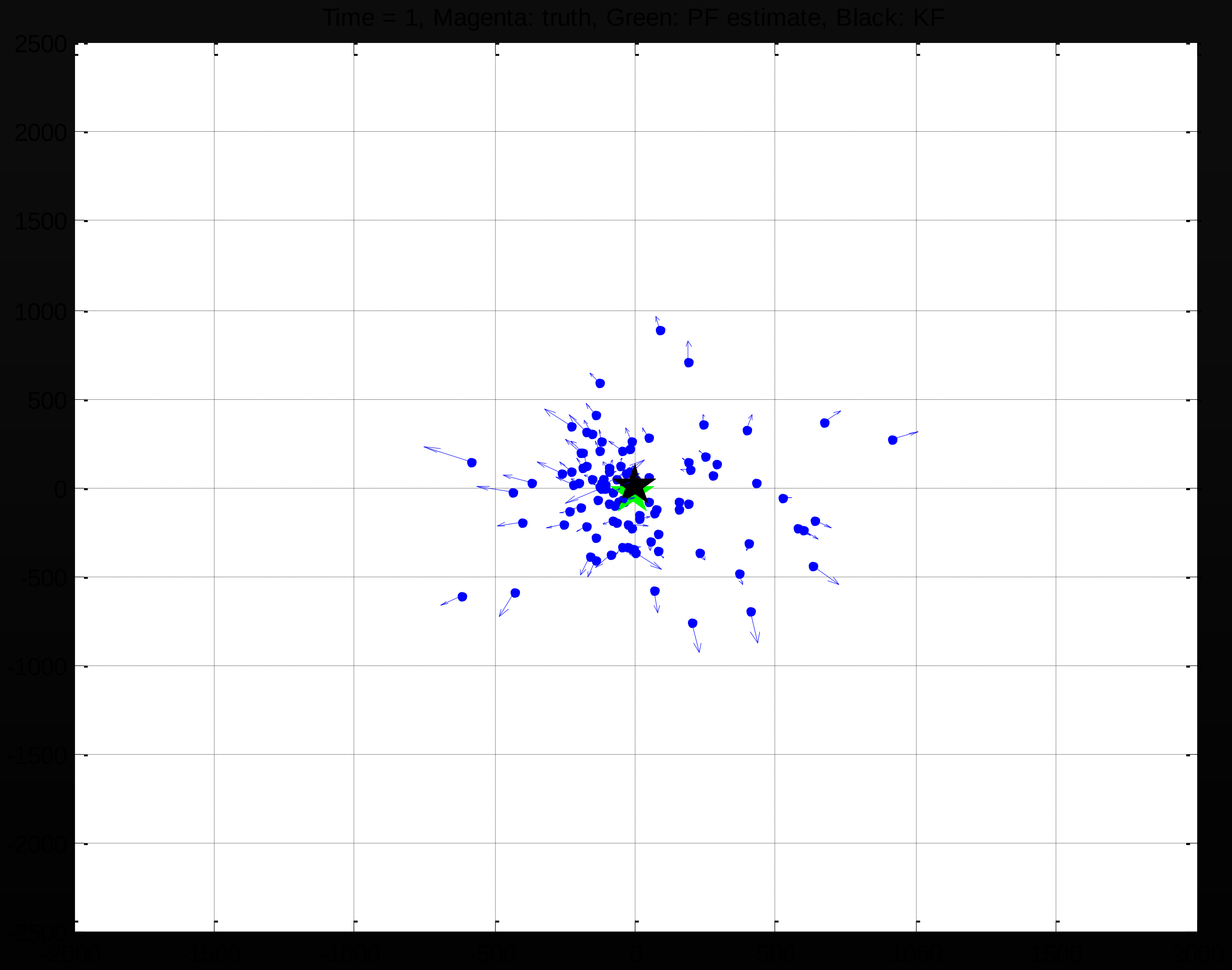
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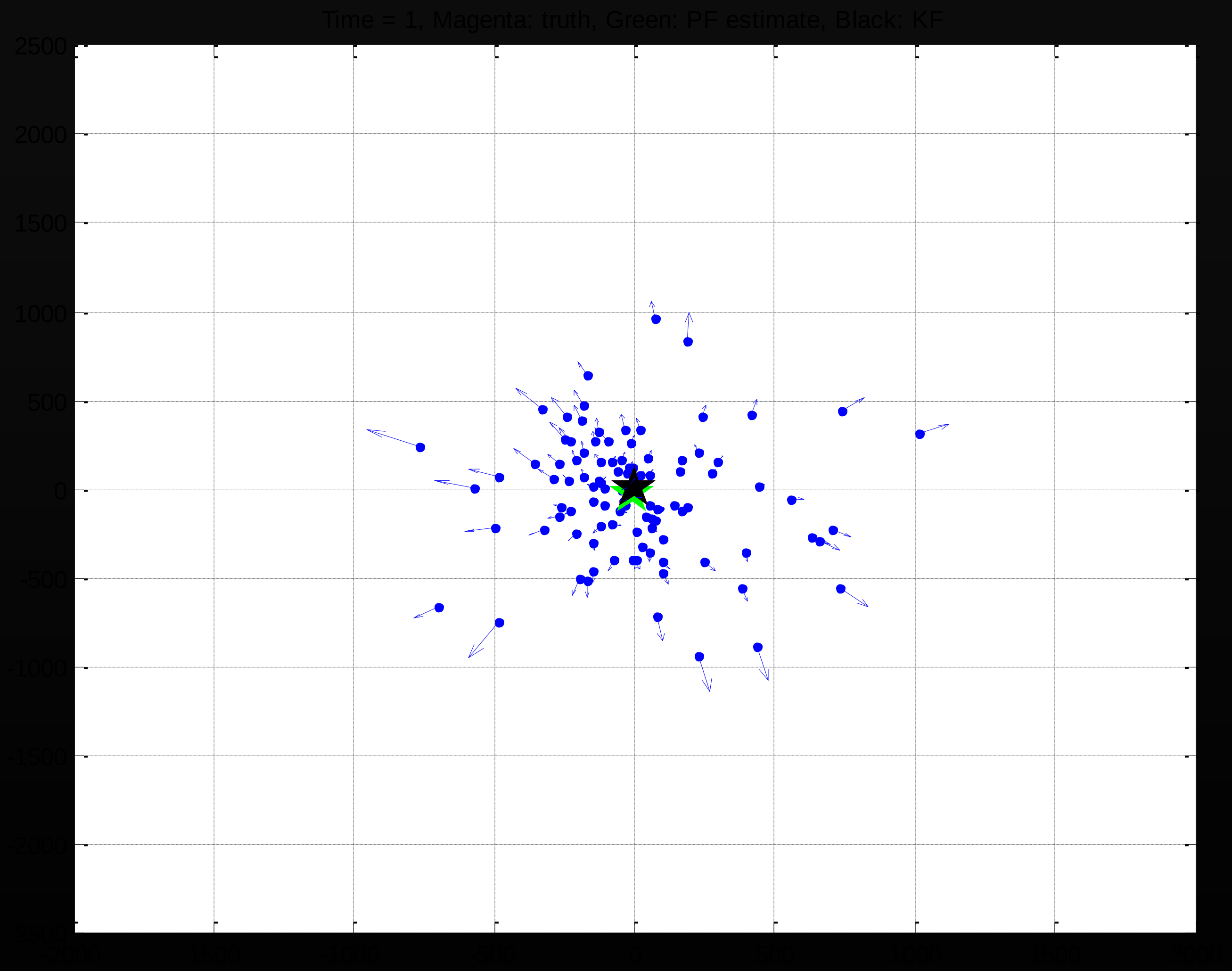
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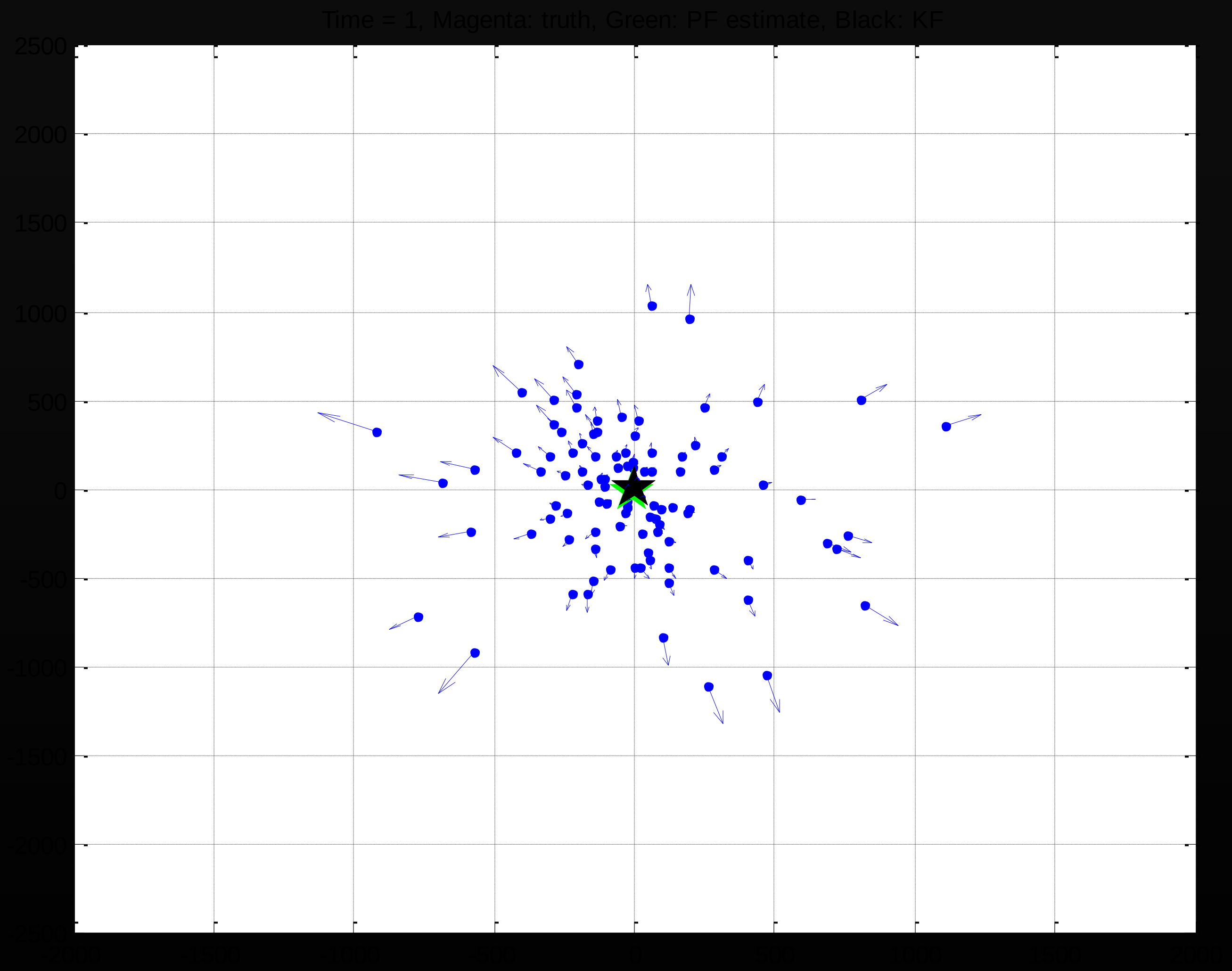
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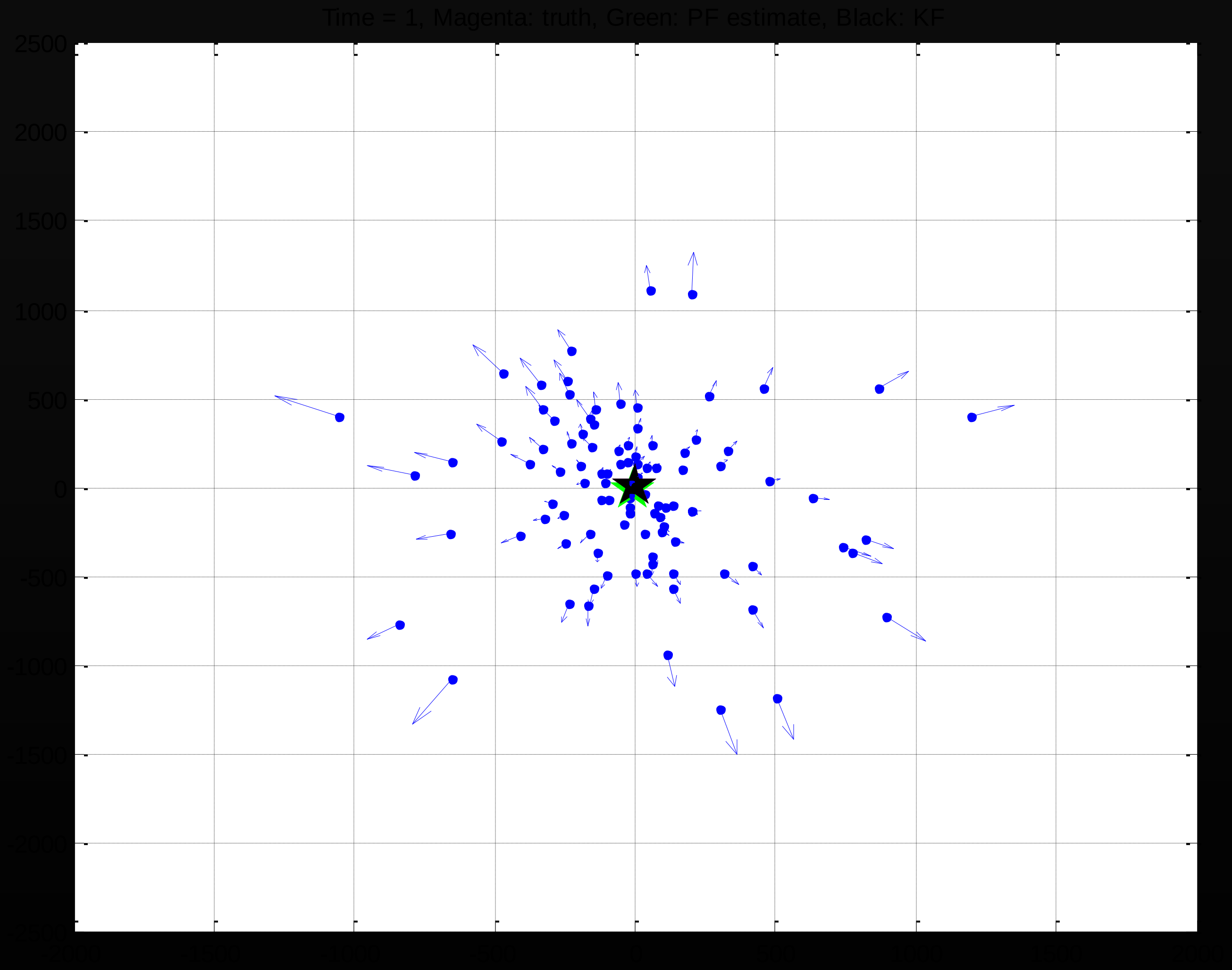
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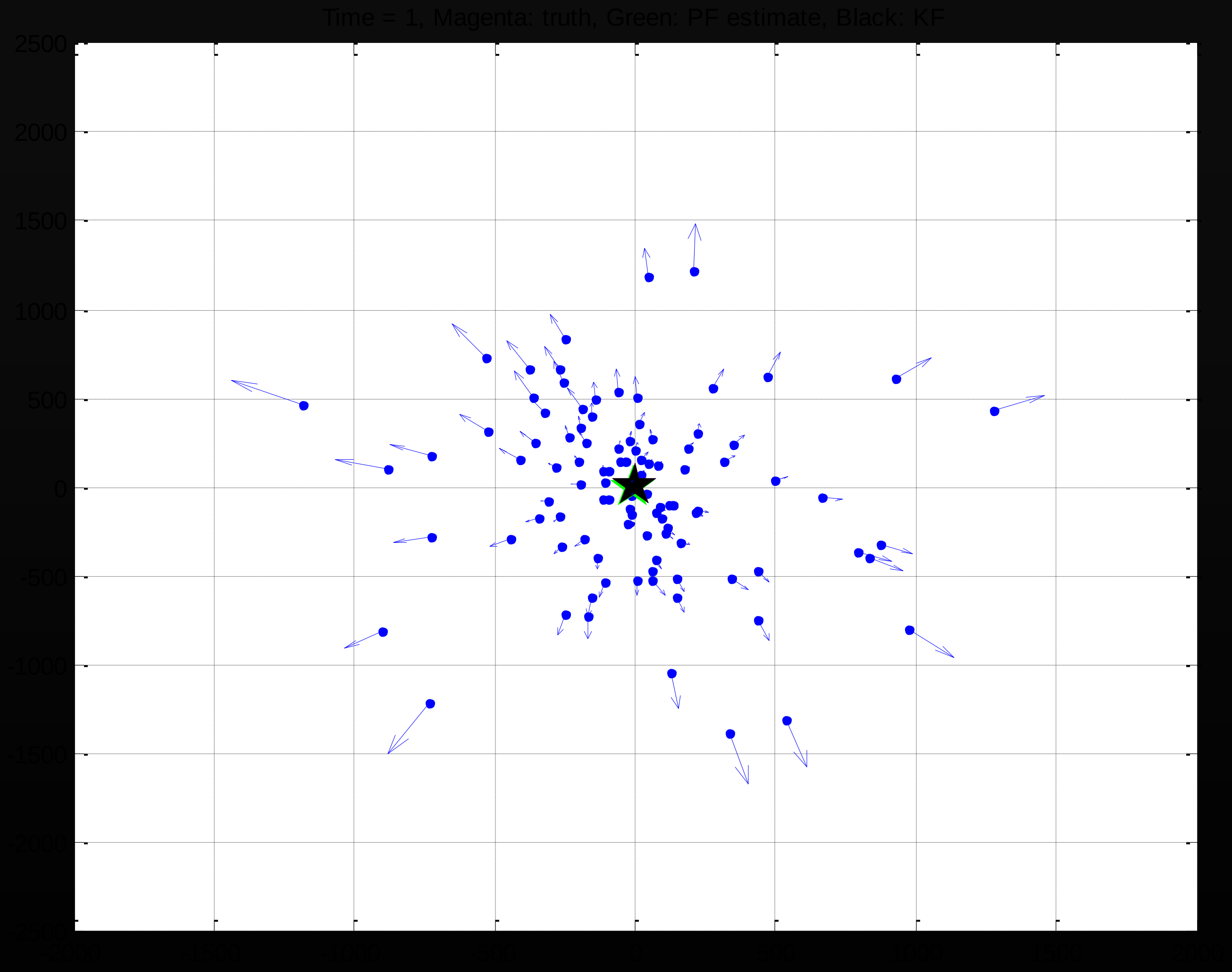
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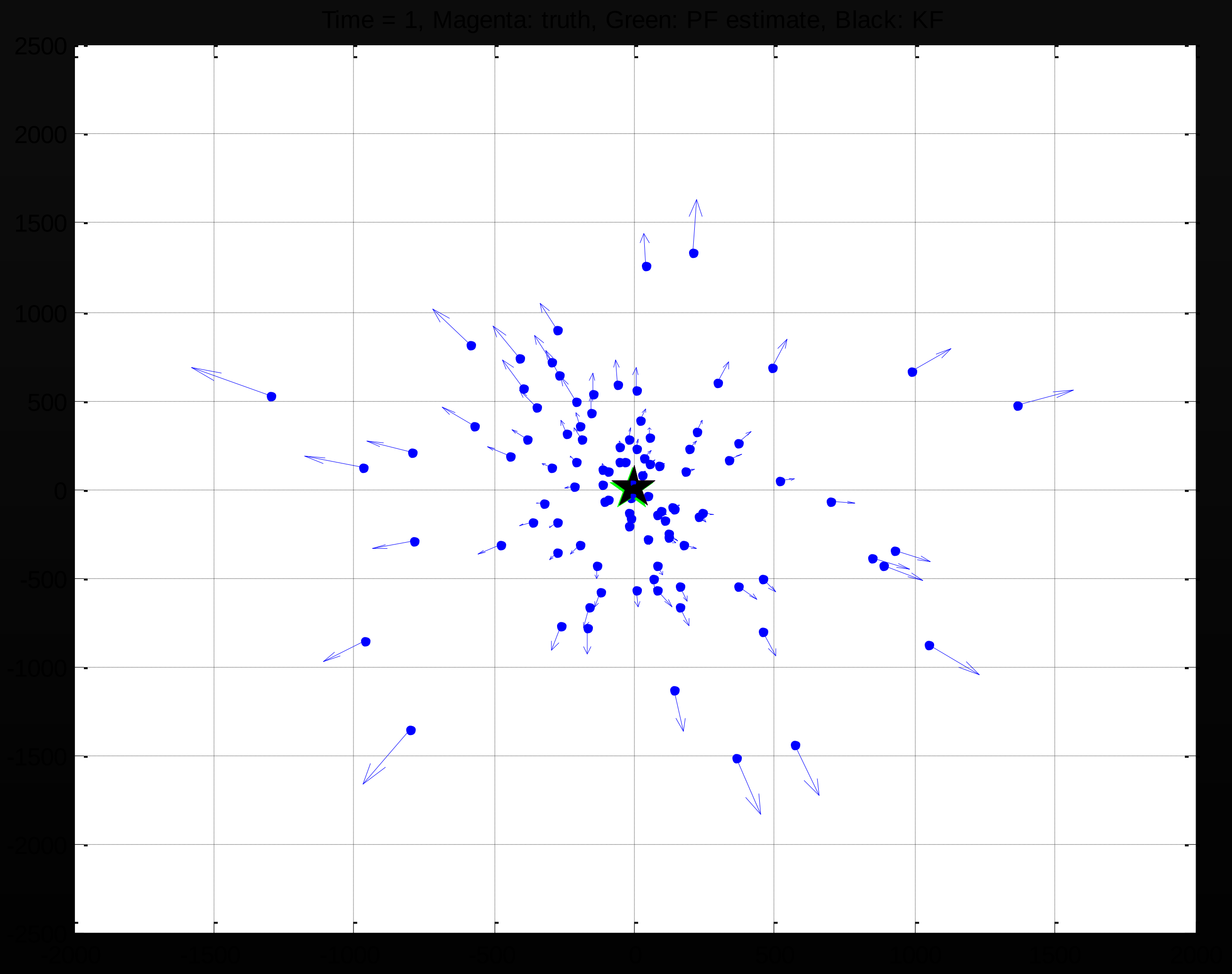
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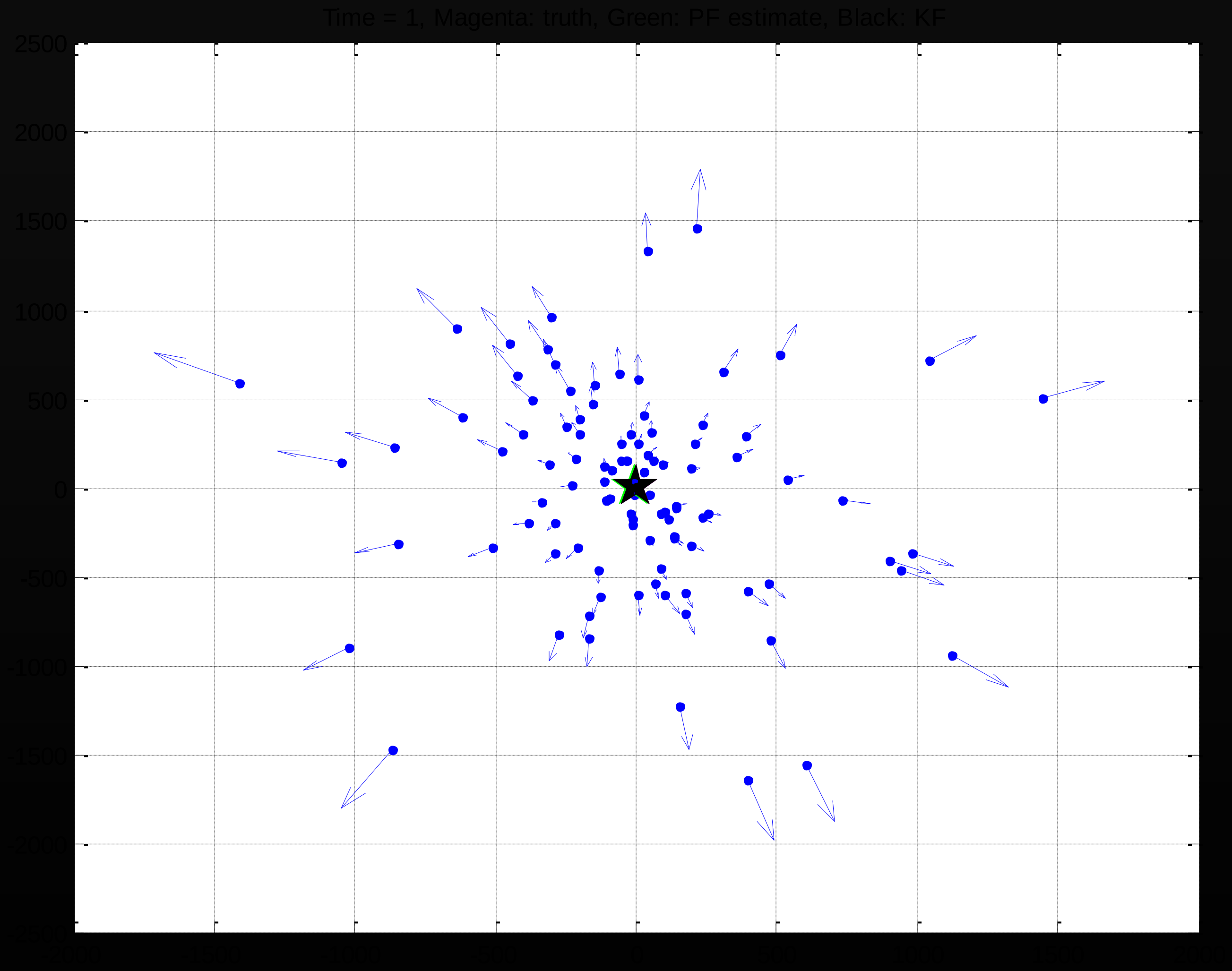
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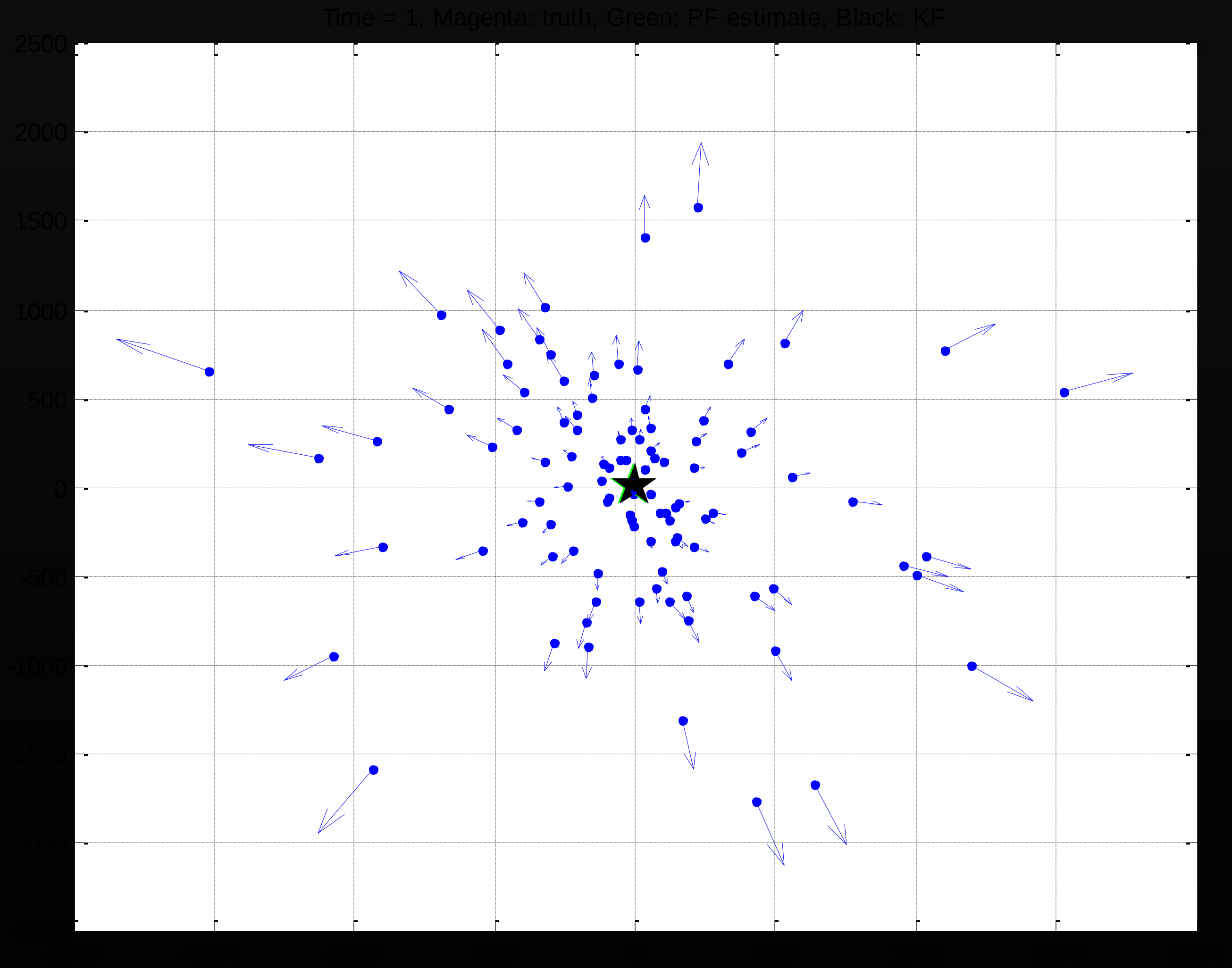
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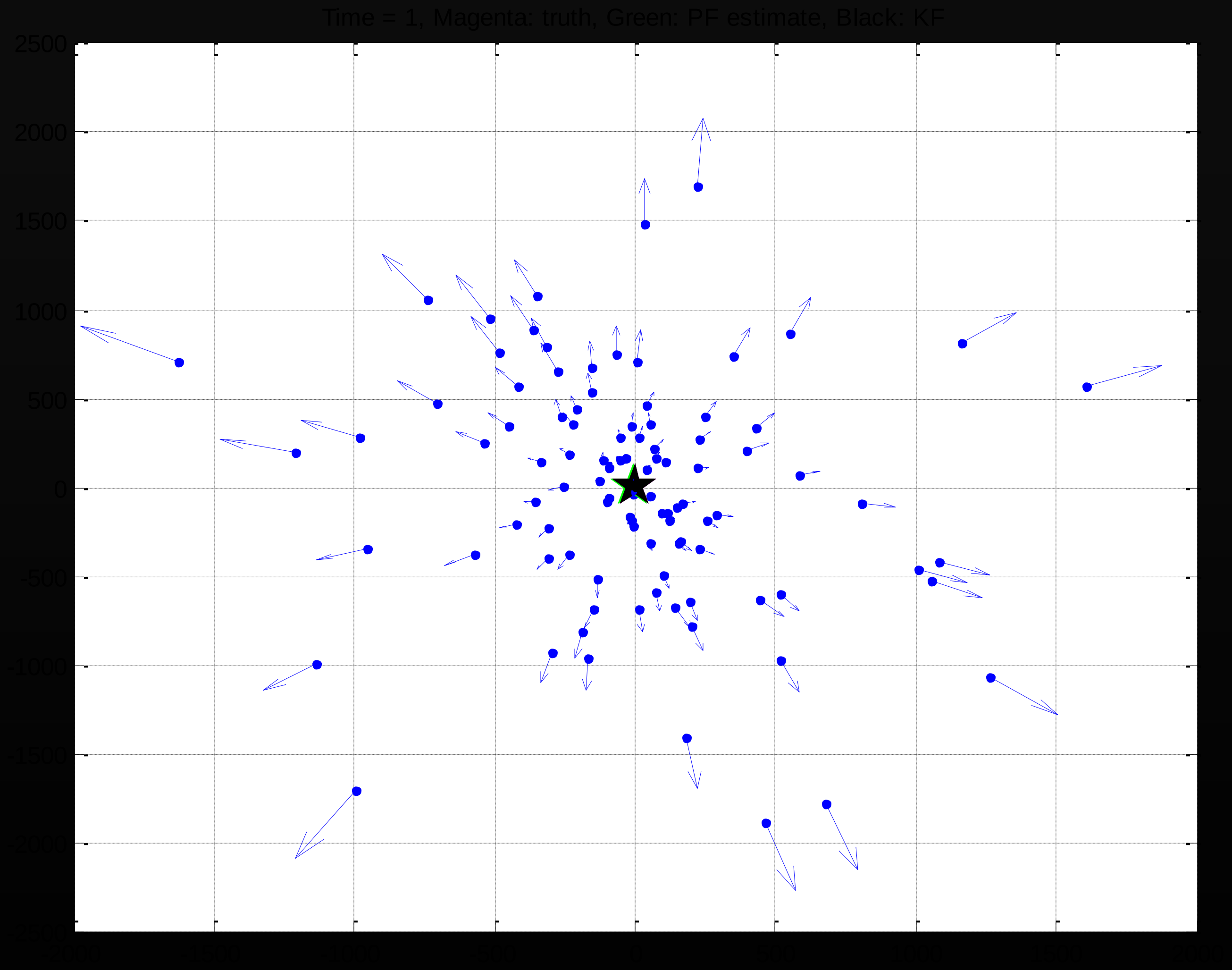
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$$div(pf) = \left[\frac{d\log K(\lambda)}{d\lambda} - \log h(x)\right] p(x,\lambda)$$

$$\frac{dx}{d\lambda} = f(x,\lambda) = \left[\frac{\partial V(x,\lambda)}{\partial x}\right]^{T} / \, p(x,\lambda)$$

$$Tr\left[\frac{\partial^2 V(x,\lambda)}{\partial x^2}\right] = \left[\frac{d\log K(\lambda)}{d\lambda} - \log h(x)\right] p(x,\lambda) = \eta$$

$$q(z) = -\frac{\ddot{R}}{RH^2} \quad \text{by definition of "cosmic deceleration parameter"}$$

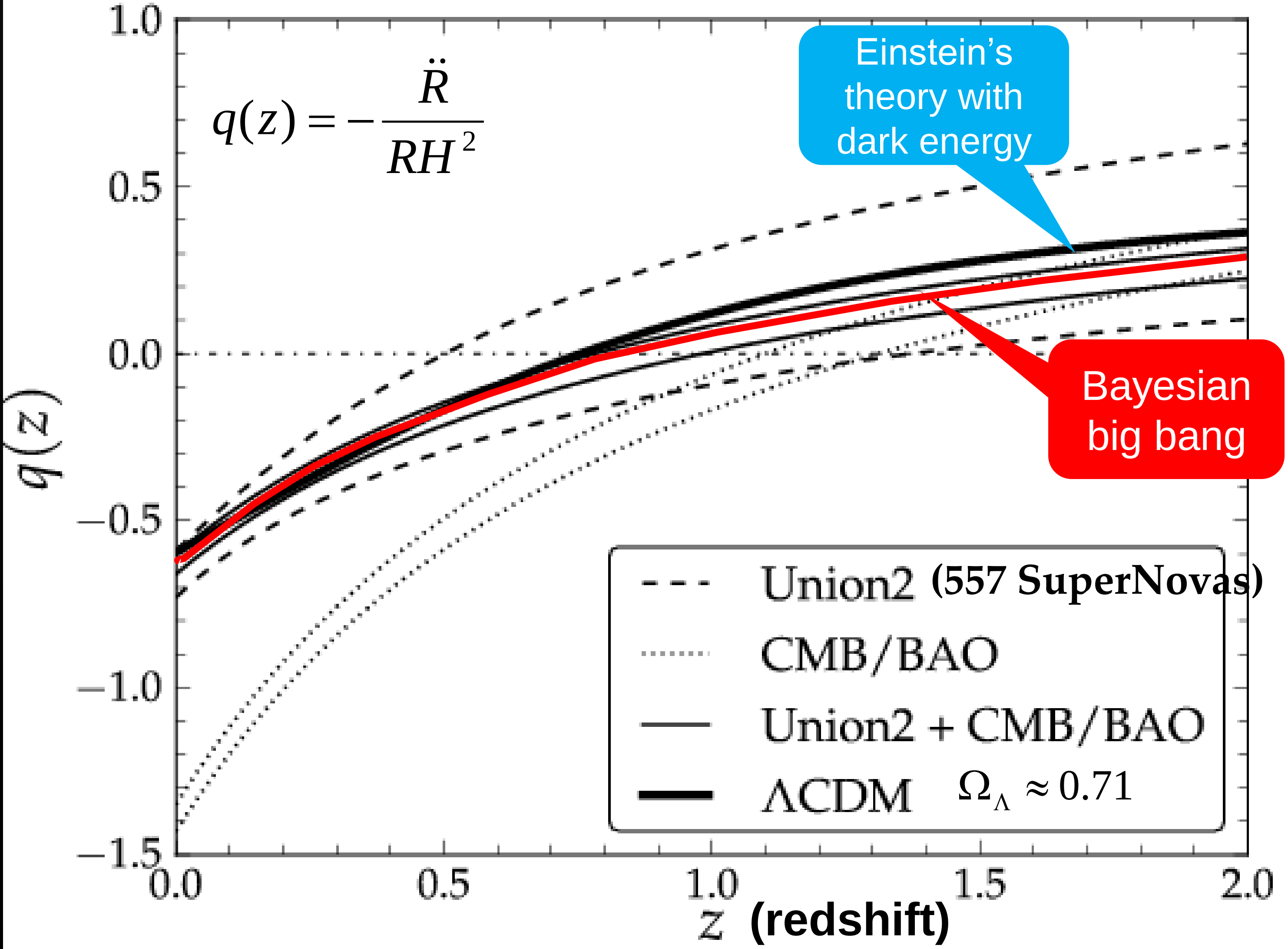
$$q(z) \approx -\eta / p = [\log h - E(\log h)] \quad \text{derived from Bayesian flow of particles}$$

$$q(z) \approx \frac{1}{2}\left[1 - \frac{\Delta m^2}{\sigma^2}\right] \quad \text{assuming Gaussian likelihood (h)}$$

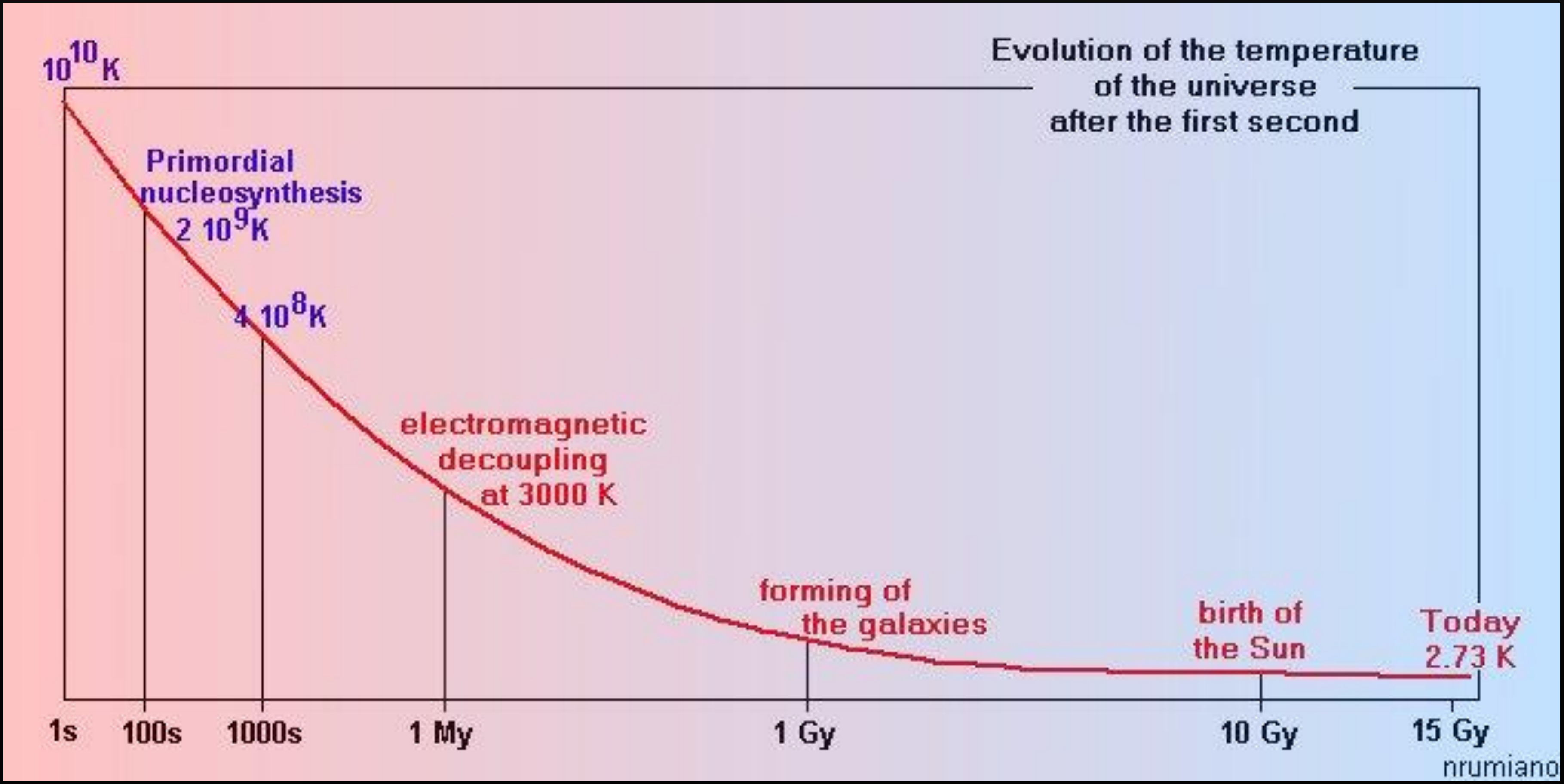
$$\sigma^2 \approx kTB\tau \quad \text{assuming thermal measurement noise}$$

assume that $\mathrm{T} \approx \mathrm{T}_0(1+\mathrm{z})$ and let $\Delta \mathrm{m}^2 = E$

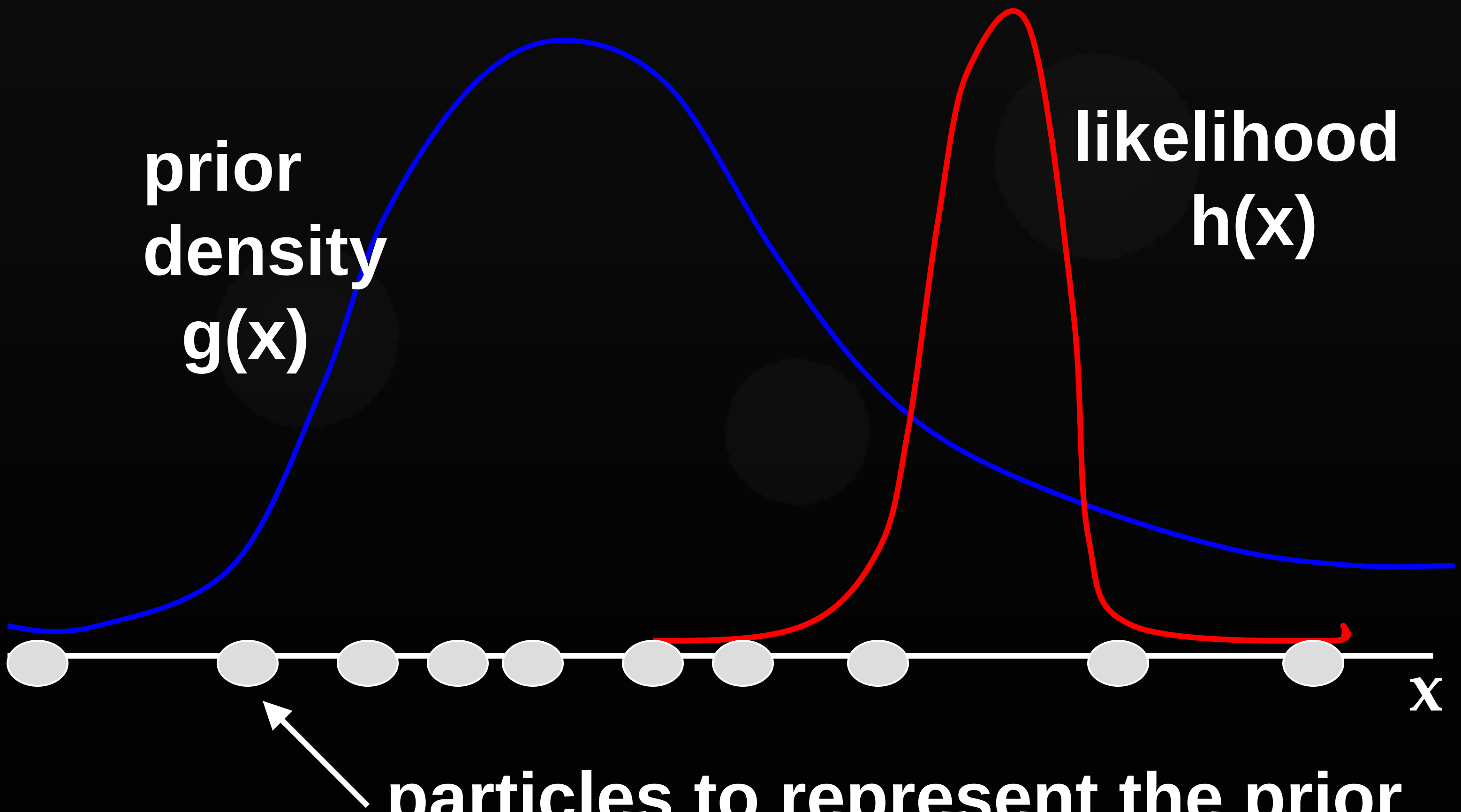
$$q(z) \approx \frac{1}{2}\left[1 - \frac{c}{(1+z)}\right] \quad \text{where } \mathrm{c} = \mathrm{E/kT_0B}\tau$$



B. Santos, et al. (Nov 2013) arXiv:1009.2733 [astro-ph.CO]

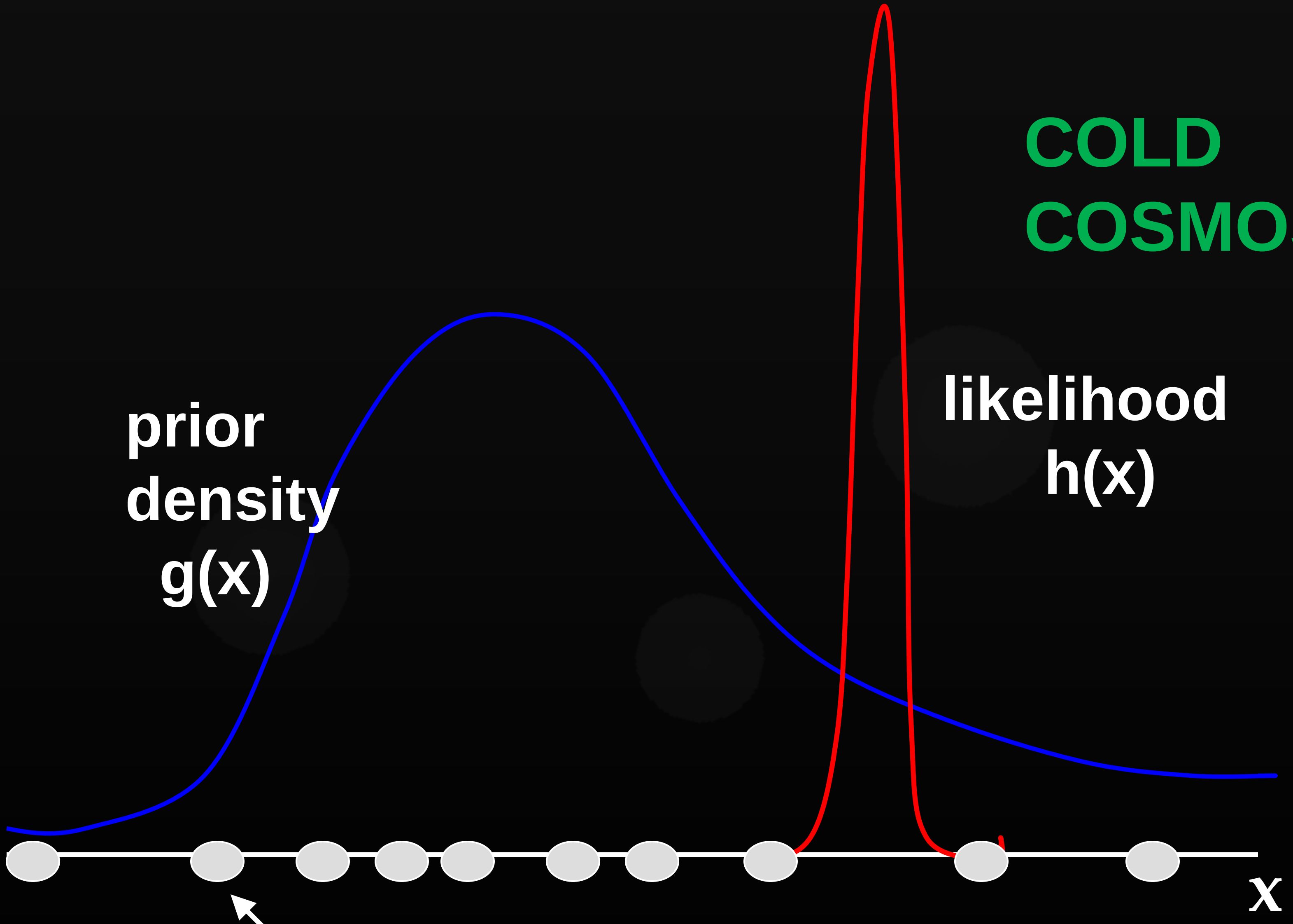

Evolution of the temperature
of the universe
after the first second
10^{10} K
Primordial
nucleosynthesis
2 10^9 K
4 10^8 K
electromagnetic
decoupling
at 3000 K
forming of
the galaxies
birth of
the Sun
Today
2.73 K
1s
100s
1000s
1 My
1 Gy
10 Gy
15 Gy
nrumiano

HOT COSMOS



particles to represent the prior

COLD
COSMOS



prior
density
g(x)

likelihood
h(x)

x

particles to represent the prior

$$q(z) \approx \frac{1}{2}\left[1 - \frac{c}{(1+z)}\right]$$

(1) The physical cause of late time cosmic acceleration is the cooling of the cosmos in our Bayesian theory.

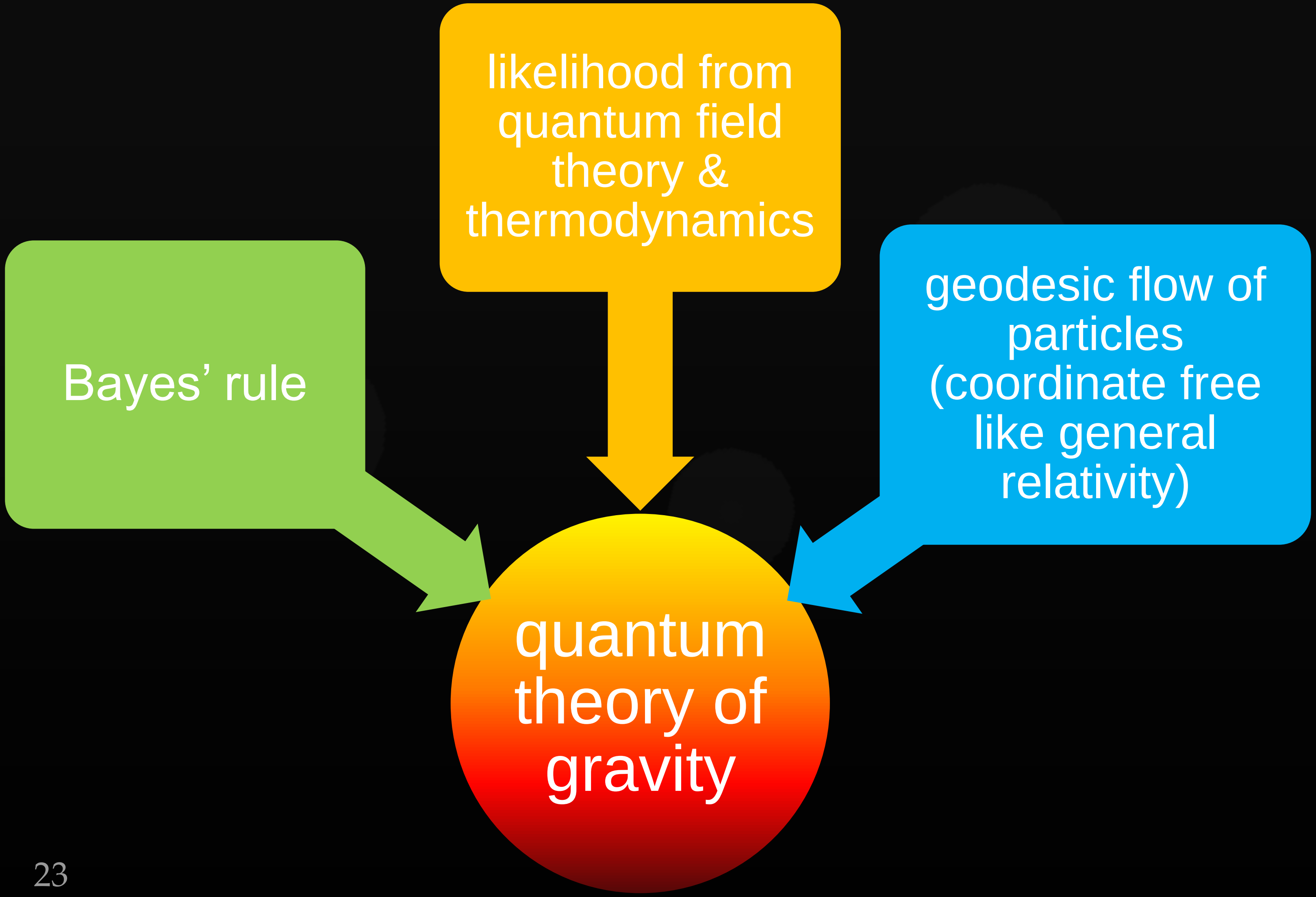
(2) Bayesian theory is in excellent agreement with cosmological data (and it agrees exactly with Friedmann matter dominated early cosmos for large z) using only one free parameter.

(3) No need for cosmological constant in general relativity, and hence no need for dark energy or 54 to 122 orders of magnitude discrepancy between cosmological data vs. quantum field theory.

(4) Related (but very different) theories by Jacobson (1995), Verlinde (2010), Padmanabhan (2010) & Smoot et al. (2010) using entropy & the Unruh effect but no use of Bayes’ rule.

(5) More accurate cosmological data are needed to falsify these theories, and such experiments are planned.

(6) Maybe the Gauss law of particle flow is related to ‘t Hooft’s holographic principle via the Gauss theorem?



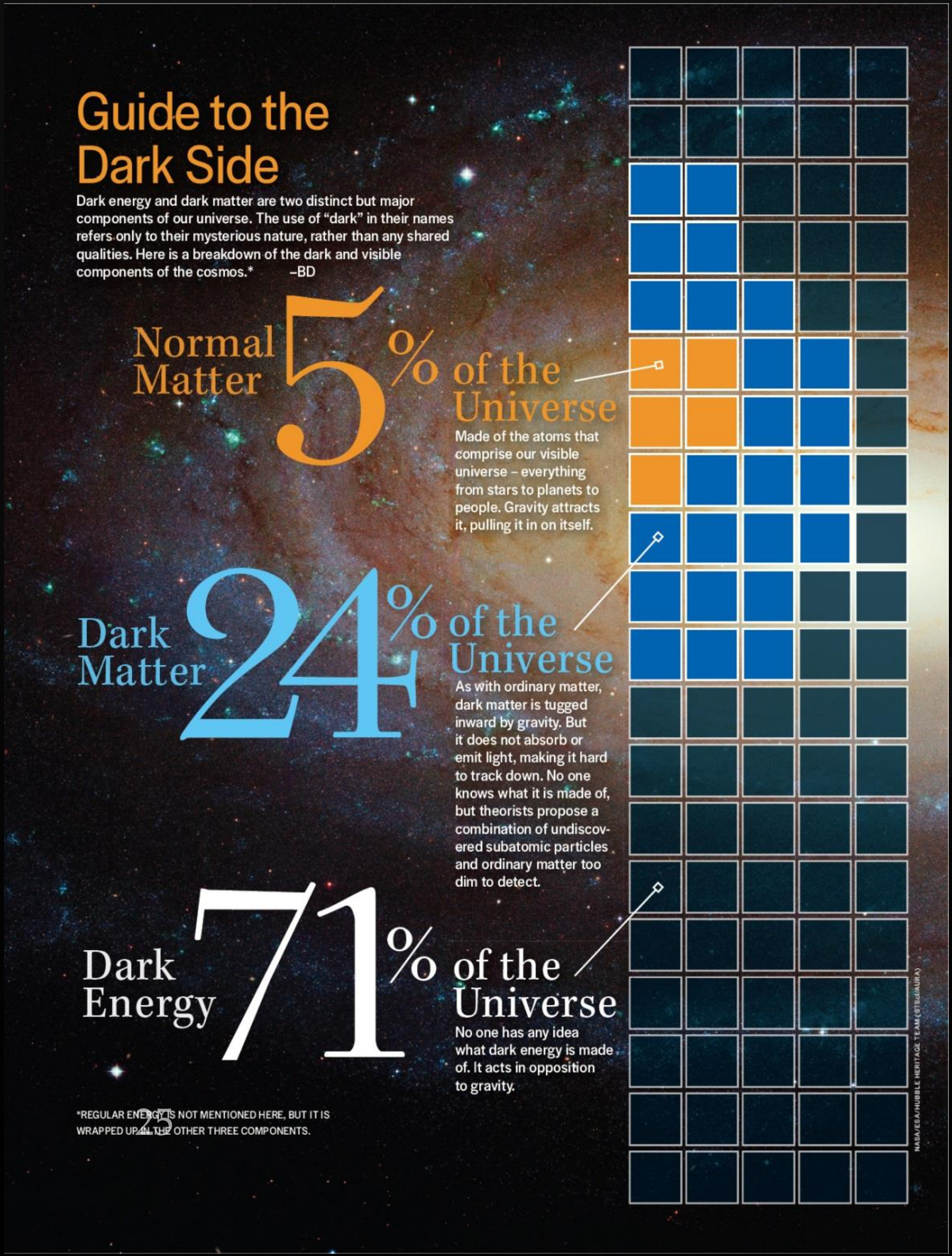
Bayes' rule

likelihood from quantum field theory & thermodynamics

geodesic flow of particles (coordinate free like general relativity)

quantum theory of gravity

BACKUP



Guide to the Dark Side

Dark energy and dark matter are two distinct but major components of our universe. The use of "dark" in their names refers only to their mysterious nature, rather than any shared qualities. Here is a breakdown of the dark and visible components of the cosmos.* –BD

Normal Matter 5% of the Universe

Made of the atoms that comprise our visible universe – everything from stars to planets to people. Gravity attracts it, pulling it in on itself.

Dark Matter 24% of the Universe

As with ordinary matter, dark matter is tugged inward by gravity. But it does not absorb or emit light, making it hard to track down. No one knows what it is made of, but theorists propose a combination of undiscovered subatomic particles and ordinary matter too dim to detect.

Dark Energy 71% of the Universe

No one has any idea what dark energy is made of. It acts in opposition to gravity.

*REGULAR ENERGY IS NOT MENTIONED HERE, BUT IT IS WRAPPED UP IN THE OTHER THREE COMPONENTS.

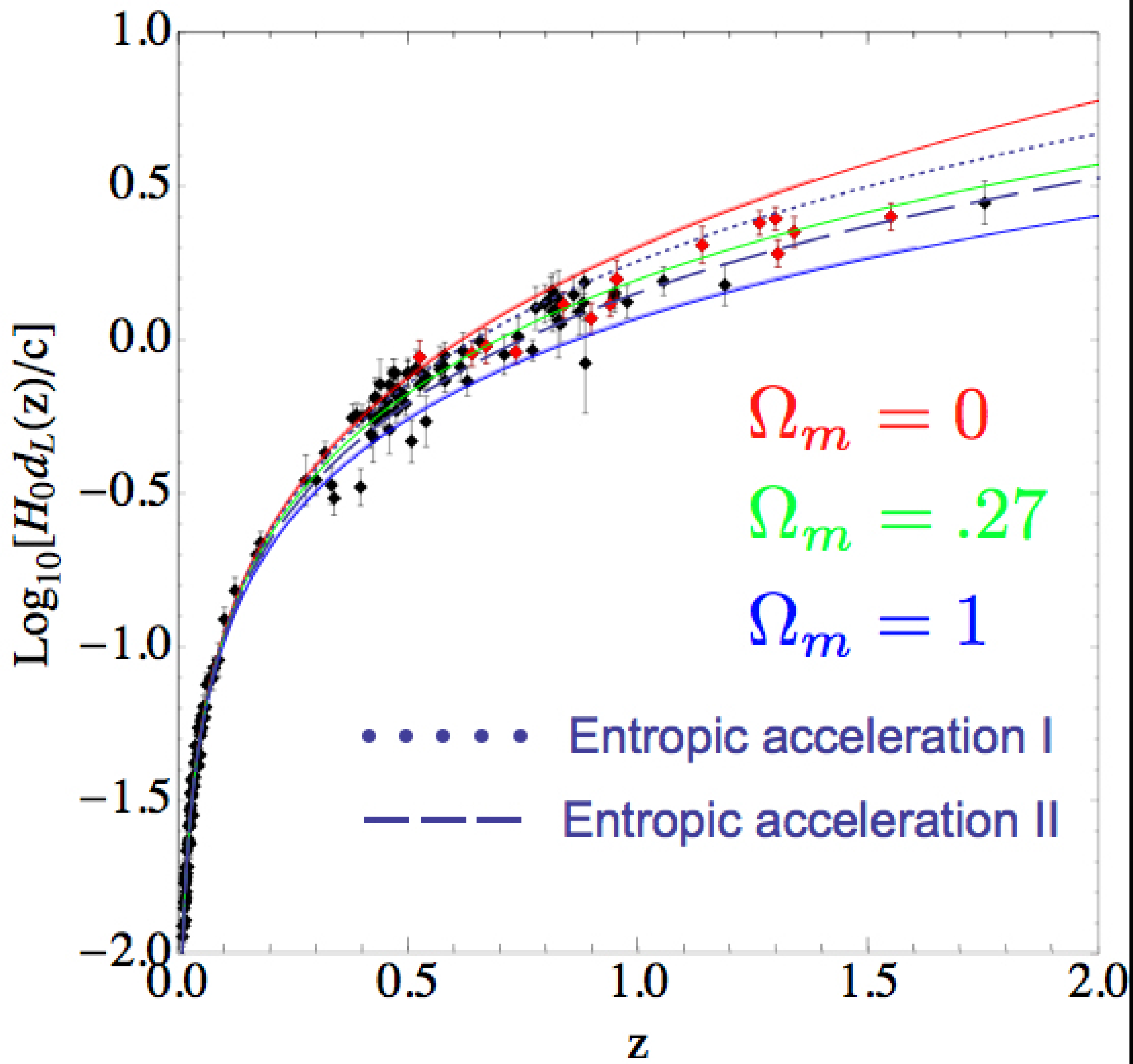
NASA/ESA/HUBBLE HERITAGE TEAM (STScI/AURA)

" 'Most embarrassing observation in physics' – that's the only quick thing I can say about dark energy that's also true." Edward Witten

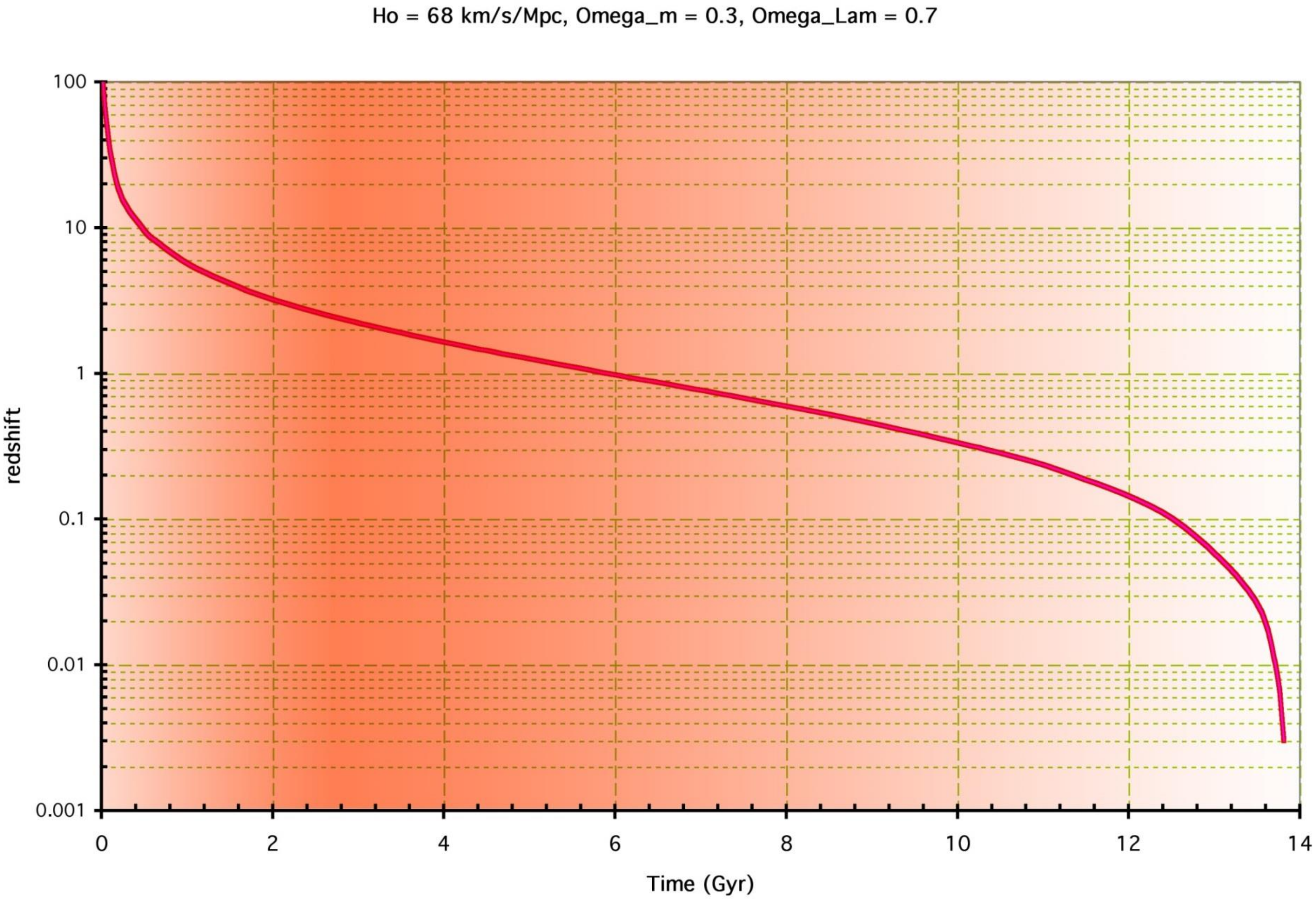
$$\text{data} = 7\text{x}10^{-30}\ \text{g/cm}^3$$

$$\text{theory} = 10^{93}\ \text{g/cm}^3$$

"Something is clearly seriously wrong."
Roger Penrose



Easson, Frampton & Smoot, "entropic accelerating universe," October 2010.


Ho = 68 km/s/Mpc, Omega_m = 0.3, Omega_Lam = 0.7
redshift
100
10
1
0.1
0.01
0.001
0
2
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6
8
10
12
14
Time (Gyr)

THEORY	FITS DATA?	NUMBER OF FREE PARAMETERS?	IDEA	NEEDS DARK ENERGY TO EXPLAIN COSMIC ACCELERATION	NEGATIVE ENERGY? OR NEGATIVE TEMPERATURE? OR UNSTABLE?	RELATIVISTIC ?
1. Einstein (1916)	yes*	at least 3 or 4	equivalence principle & Riemannian space-time & Newtonian weak field limit	yes	no	yes
2. Dicke-Brans (1961)	yes*	at least 4 or 5	non-constant G & Mach's principle	yes	no	yes
3. Milgrom (1983)	no	3 per galaxy	ad hoc lower limit on Newton's gravity	yes	no	no
4. Beckenstein & Sanders (2004)	no	many	extra scalar & vector fields	yes	yes	no
5. Moffat (1995 & 2004 & 2009)	yes!	at least 8	asymmetric metric tensor & extra fields	no	no	yes
6. Jacobson (1995)	yes	zero	entropic force & Unruh effect	no	yes ?	yes
7. Padmanabhan (2010)	yes	zero	max entropy & Unruh effect	no	yes ?	yes
8. Verlinde (2010)	yes	zero	holographic principle, entropic force & Unruh effect	no	yes	yes
9. Easson, Frampton & Smoot (2010)	yes	zero	entropic force & Unruh effect	no	?	yes
10. Daum (2011)	yes	one	Bayes' rule	no	no	no

Bayesian physics

(1) Sanjoy Mitter & Nigel Newton, “information and entropy flow in Kalman-Bucy filter,” 2004.

(2) Sean Carroll, et al., “the Bayesian 2nd law of thermodynamics,” August 2015.

(3) Hrant Gharibyan & Max Tegmark, “sharpening the 2nd law of thermodynamics with the quantum Bayes theorem,” Physics Rev E, 2014.

(4) Jonathan Heckman, “statistical inference and string theory,” July 2013.

(5) Brad Chase, “parameter estimation, model reduction and quantum filtering,” thesis 2009.

(6) Mankei Tsang, “Ziv-Zakai error bounds for quantum parameter estimation,” 2012.

reference	comments
1. Roger Balian, et al., “lectures on dynamical models for quantum measurements” June 2014.	derivation of Born rule from a dynamical model of coupled macroscopic system with quantum system; solution of the so-called “measurement problem” by using standard probability theory (for the joint system with non-equilibrium quantum thermodynamics)
2. Klaus Hornberger, “introduction to decoherence theory,” 2009.	concrete explicit physical examples of quantum master equation; excellent tutorial
3. Bengt Svensson, “A pedagogical review of quantum measurement theory,” Quanta May 2013.	derivation of quantum master equation with explicit measurement model (double bracket form) like Zakai equation in nonlinear filter theory; also derivation of the Leggett-Garg inequality with weak measurements
4. Kurt Jacobs and Daniel Steck, “a straightforward introduction to continuous quantum measurement,” contemporary physics, 2006.	accessible tutorial on continuous time quantum measurements, with a derivation of the stochastic master equation conditioned on measurements
5. Mankei Tsang, “time-symmetric quantum theory of smoothing,” 2009.	derivation of quantum Zakai equation for smoothing assuming weak measurements & linear-Gaussian problem
6. Mankei Tsang, “Ziv-Zakai error bounds for quantum parameter estimation,” 2012.	generalized Cramér-Rao bound & Heisenberg uncertainty principle for repeated noisy quantum measurements
7. “quantum theory & measurement,” edited by Wheeler & Zurek, 1983.	ancient history (collection of classic papers on the so-called “quantum measurement problem” up to 1983)

reference	comments
8. Brad Chase, “parameter estimation, model reduction and quantum filtering,” doctoral thesis on quantum filtering (December 2009)	simple concrete physical examples of quantum filters, with MATLAB simulation results for quantum Kalman filters & quantum particle filters; very accessible.
9. Carlos Brasil et al., “a simple derivation of the Lindblad equation” 2012.	nice simple tutorial on Lindblad operator for quantum filtering
10. Philip Pearle, “simple derivation of the Lindblad equation” [sic] 2012.	nice simple tutorial on Lindblad operator
11. Adler & Bassi, “collapse models with non-white noise” 2007	simple derivation of Lindblad equation from stochastic Schrödinger equation using Ito calculus
12. Attal & Pelligrini, “stochastic master equation in thermal environment” 2008	how to model non-zero temperature in Schrödinger equations (this is crucial)
13. Justin Dressel, “weak values are interference phenomena” 2014	nice physical explanation of anomalous values of probabilities
14. Leslie Ballentine, “quantum mechanics: a modern development” 2015 [sic]	extremely clear mathematical discussion of entanglement & erroneous notions, as well as alternative (but not popular) developments of quantum mechanics

	OLD QM	NEW QM
1. measurements	collapse of the wave function (instantaneous)	continuous time using Schrödinger-Zakai equation
2. model of macroscopic measurement of quantum system	no explicit model	explicit model in Schrödinger-Zakai equation (Lindblad operator)
3. connection between quantum mechanics & probability	Born rule	Bayes' rule
4. uncertainty bounds for quantum mechanics	Heisenberg uncertainty principle	Cramér-Rao-Heisenberg & Ziv-Zakai-Heisenberg bounds
5. entanglement & decoherence	paradoxes & confusion & counterintuitive notions	explicit equations derived from math & physics
6. propagation of wavefunction	unitary without measurements & Born's rule for measurements	non-unitary for measurements & unitary without measurements

Bayes’ rule:

$$p(x,t_k|Z_k) = \frac{p(x,t_k|Z_{k-1})\,p(z_k|x,t_k)}{p(z_k|Z_{k-1})}$$

$p(x,t_k|Z_k) =$ probability density of x at time $\mathrm{t_k}$ given $\mathrm{Z_k}$

$x =$ state vector

$\mathrm{t_k}$ = time of the $\mathrm{k^{th}}$ measurement

$\mathrm{z_k} = k^{th}$ measurement vector

$\mathrm{Z_k}$ = set of all measurements up to & including time $\mathrm{t_k}$

$$Z_k = \{z_1, z_2, z_3, \ldots, z_k\}$$

Einstein's general relativity for small ball zero initial velocity homogeneous & isotropic approximations

$$\frac{\ddot{V}}{V} \approx -4\pi G\left[\rho_m + \frac{1}{c^2}\left(P_x + P_y + P_z\right)\right]$$

$$\frac{3\ddot{R}}{R} \approx -\frac{1}{2}\left(\rho + 3P - 2\Lambda\right)$$

$$\frac{3\ddot{R}}{R} \approx -\frac{k}{2R^3} + \Lambda$$

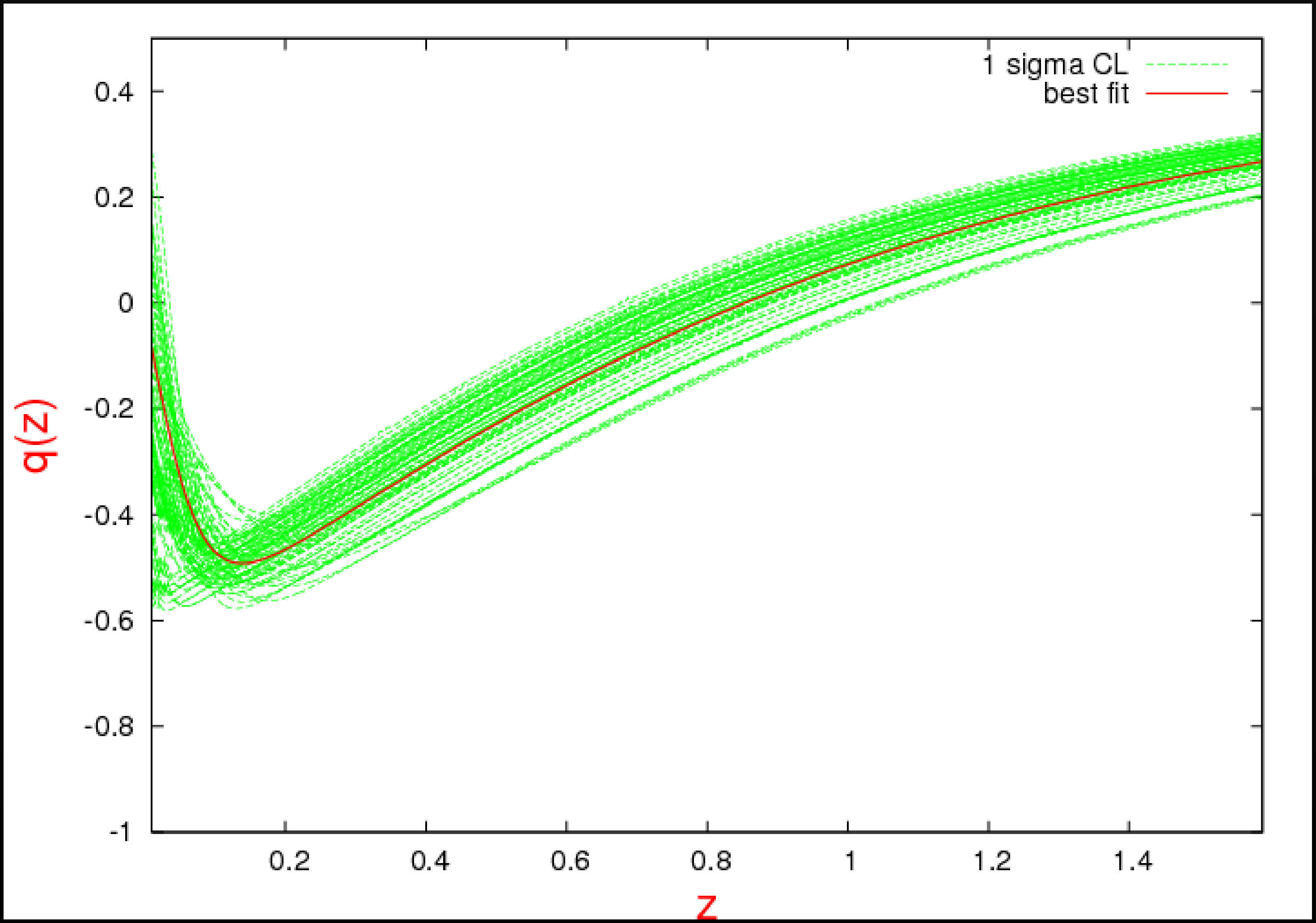
entropic force*

$$T = \frac{hH}{4\pi^2 k} \approx 3x10^{-30}\ \mathrm{K}$$

$$\mathrm{a} = \frac{4\pi^2\,\mathrm{ckT}}{h} = cH \approx 10^{-9}\ \mathrm{m/sec}^2$$

***Easson, Frampton & Smoot, "entropic accelerating universe," October 2010.**

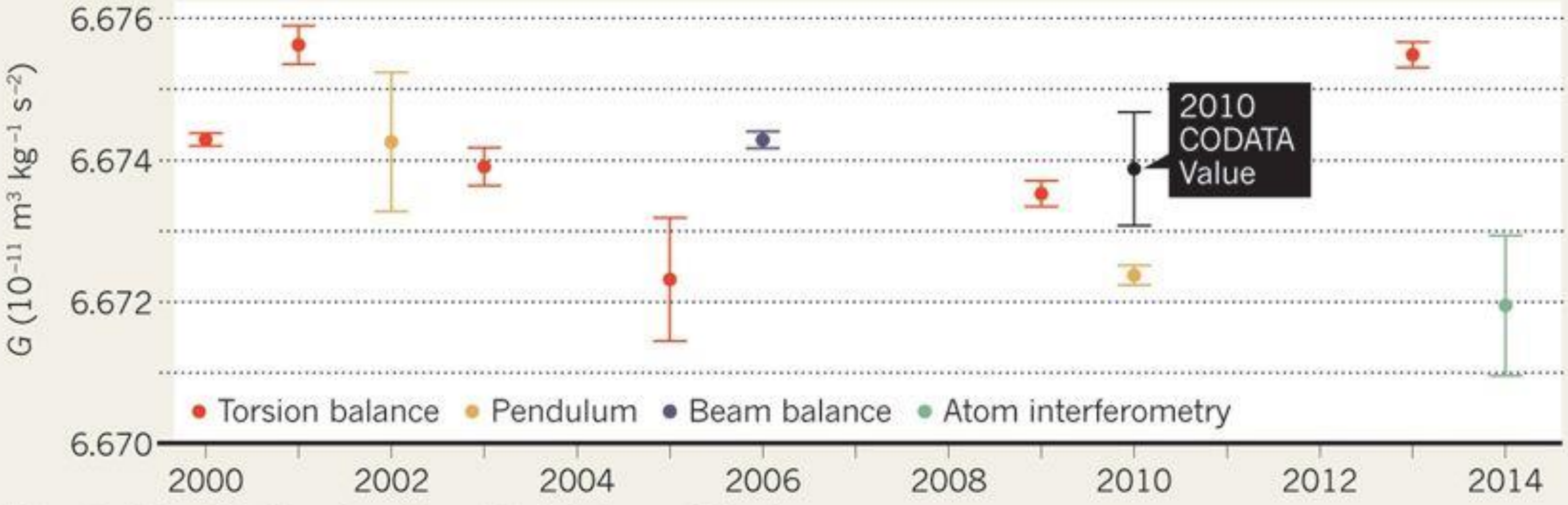
item	Bayesian big bang	Einstein's general theory of relativity
1. agrees with experimental data?	YES	YES
2. postulates existence of dark energy?	NO	YES
3. depends on thermodynamics of cosmos and quantum mechanics?	YES	NO*
4. explains big bang?	YES	NO
5. coordinate free?	NO but not needed for cosmology	YES
6. relativistic?	NO but not needed for cosmology	YES
7. consistent with QFT?	YES	NO
8. measurement model using Bayes' rule?	YES	NO
9. theory of gravity?	NO	YES
10. theory of everything?	NO	NO



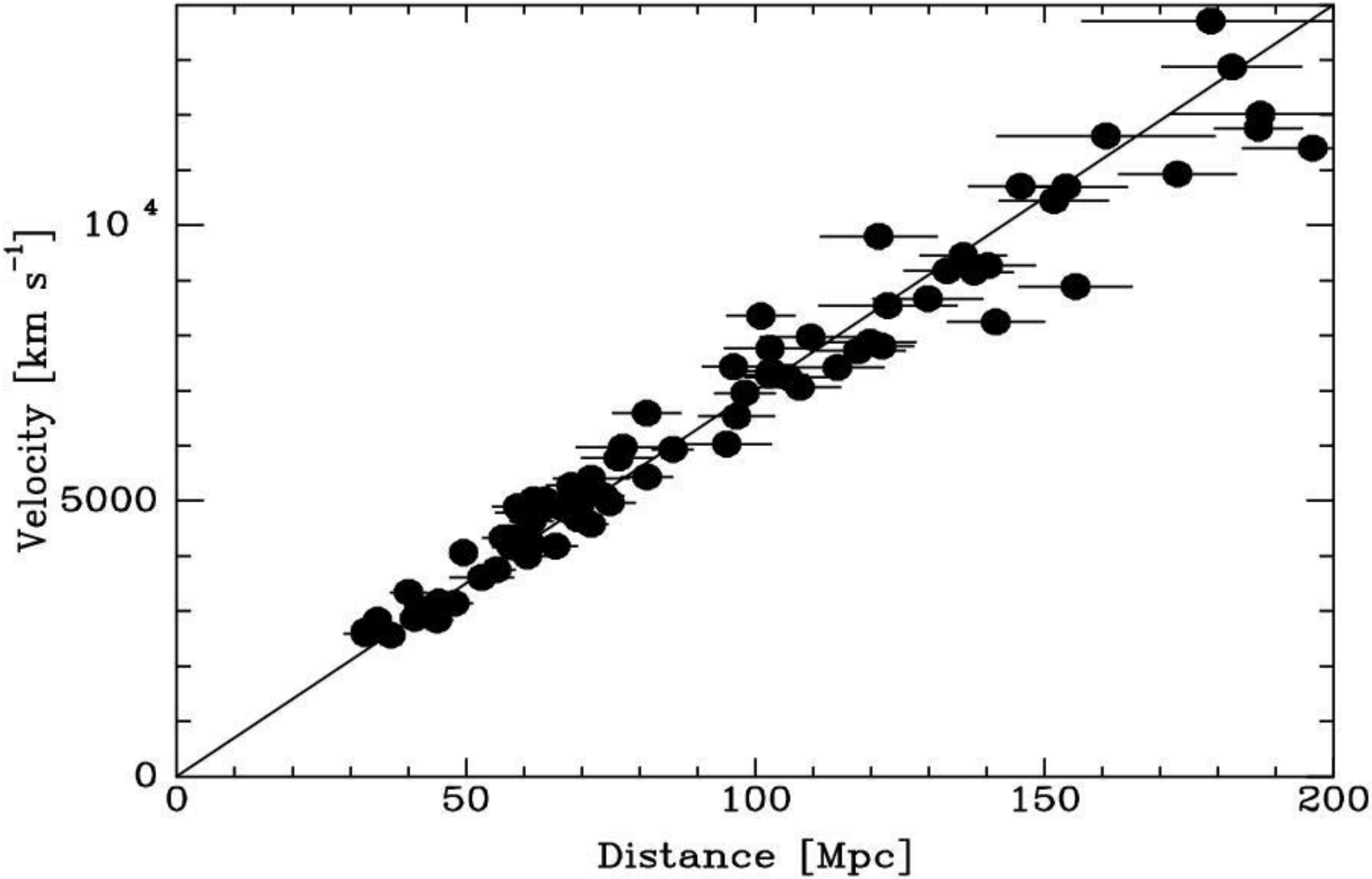
Zemets, O.A. & D.A. Yerokhin **"Cosmic acceleration a new review"** Dec 2010 arXiv:1012.2756 [astro-ph.CO]

TROUBLE WITH BIG *G*

In 2000, scientists measured the gravitational constant, *G*, with smaller error bars than ever before. But since then, a variety of experiments using different techniques have produced a range of values — and uncertainty in the official CODATA* value has increased since 2006.



*International Council for Science Committee on Data for Science and Technology


Velocity [km s^{-1}]
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50
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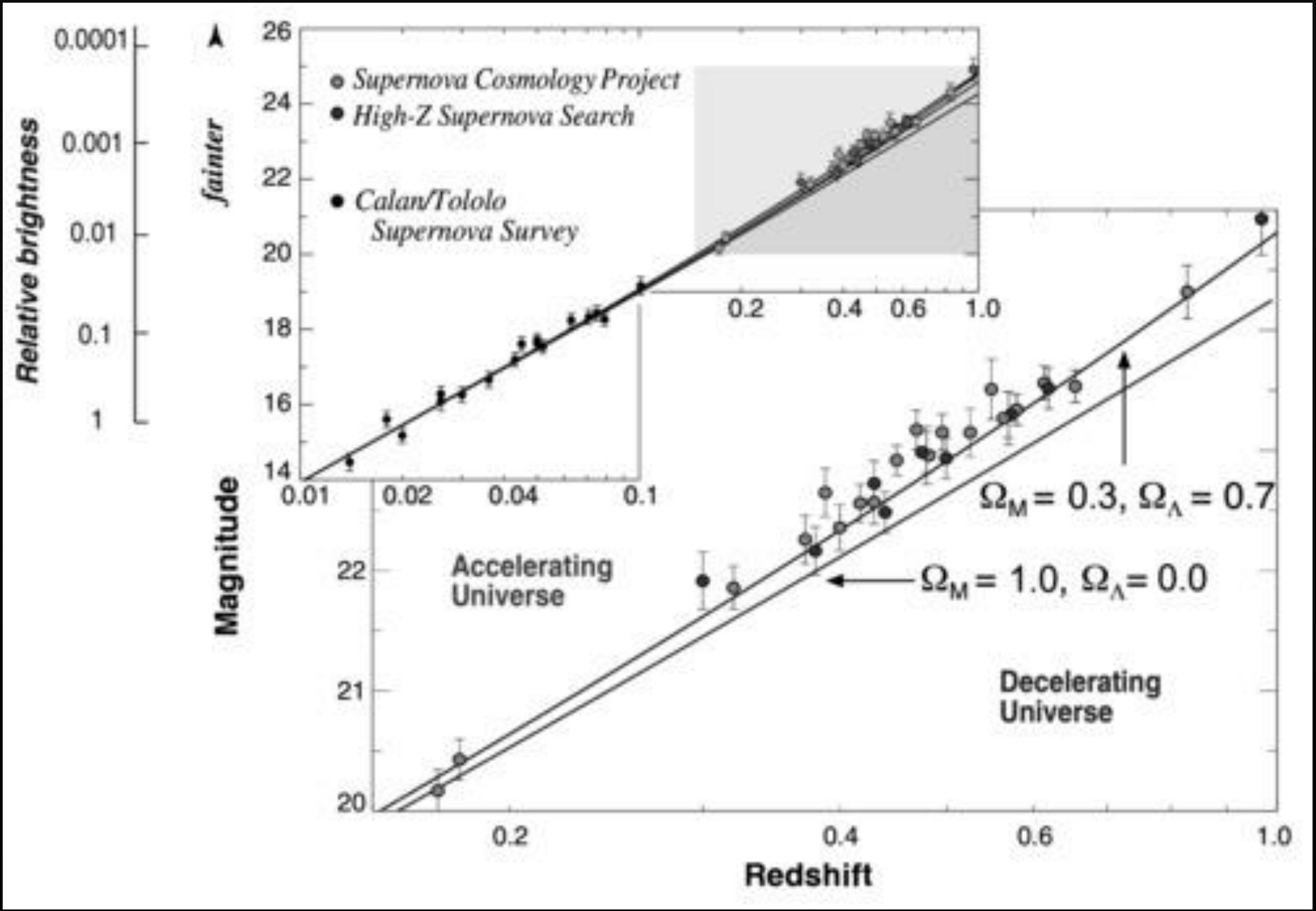
150
200
Distance [Mpc]


Relative brightness
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0.1
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22
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18
16
14
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0.02
0.04
0.1
0.2
0.4
0.6
1.0
● Supernova Cosmology Project
● High-Z Supernova Search
● Calan/Tololo Supernova Survey
Magnitude
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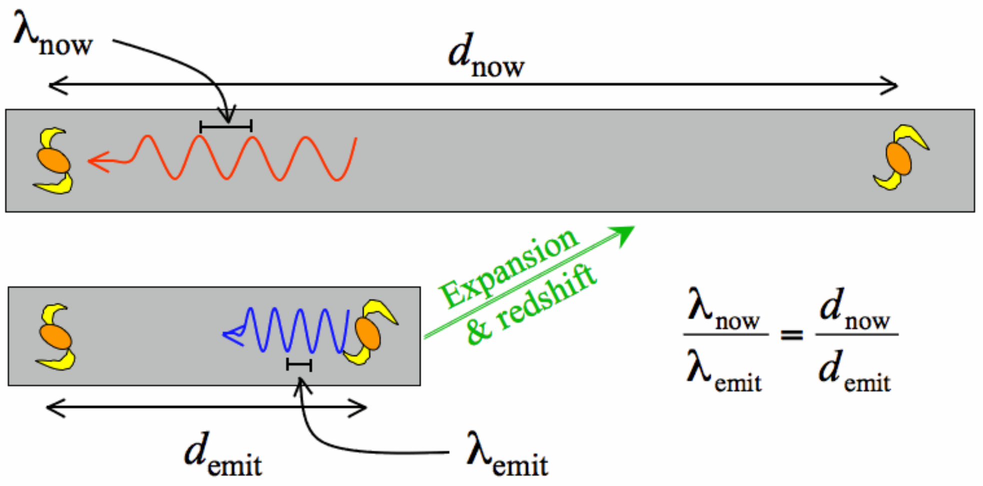
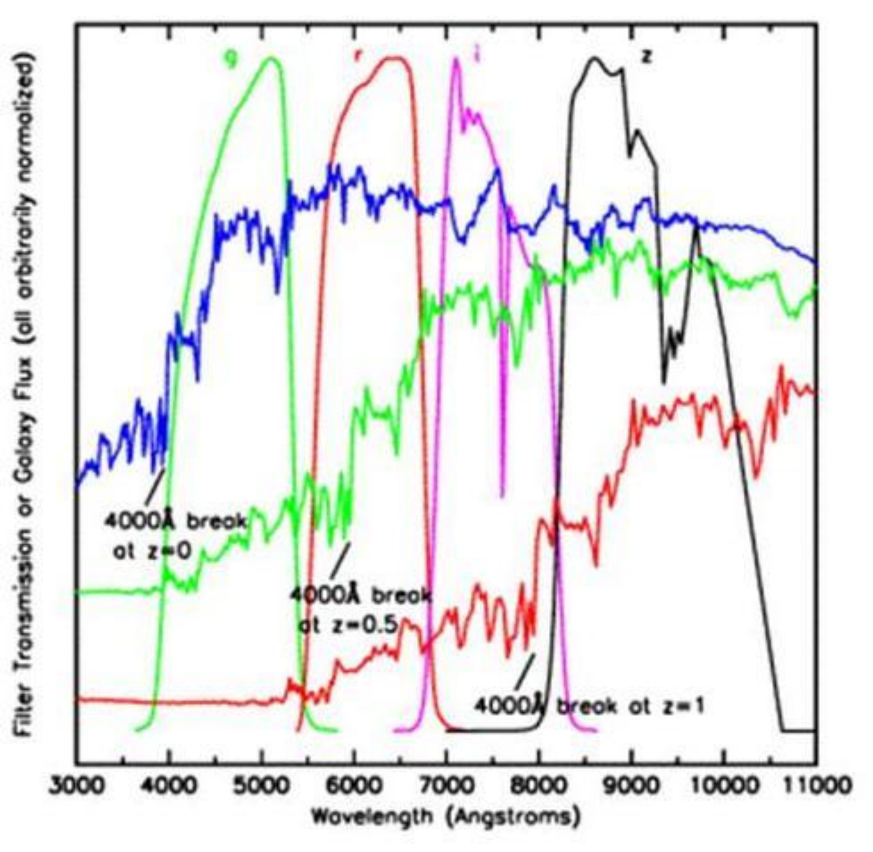
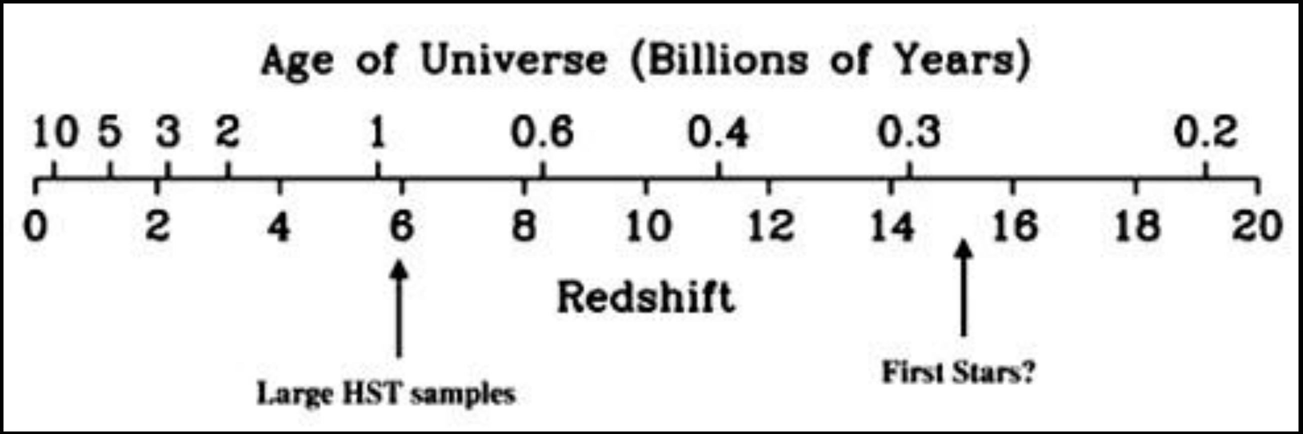
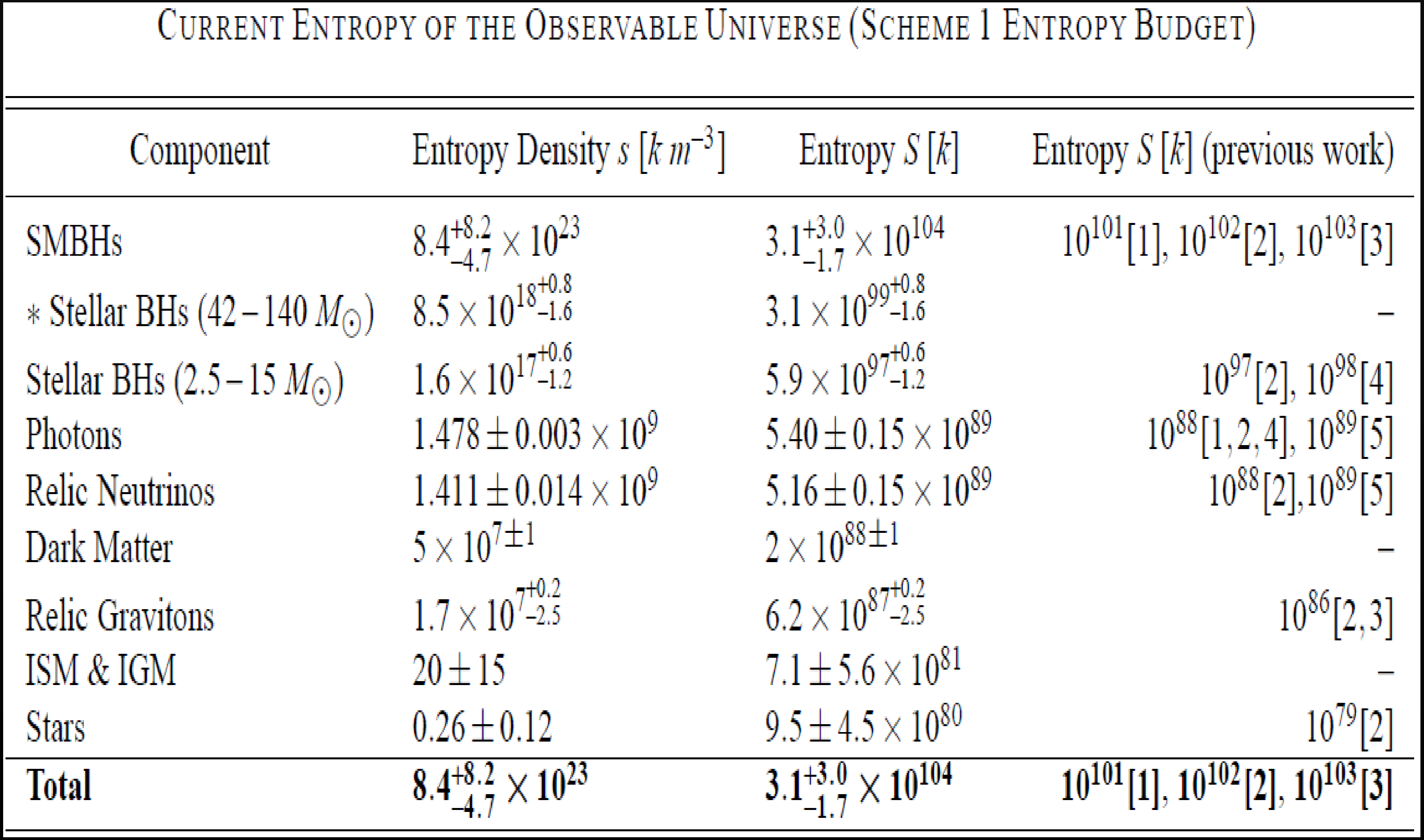
21
20
Accelerating Universe
Decelerating Universe
$\Omega_M = 0.3, \Omega_\Lambda = 0.7$
$\Omega_M = 1.0, \Omega_\Lambda = 0.0$
0.2
0.4
0.6
1.0
Redshift



$$\frac{\lambda_{now}}{\lambda_{emit}} = \frac{d_{now}}{d_{emit}}$$

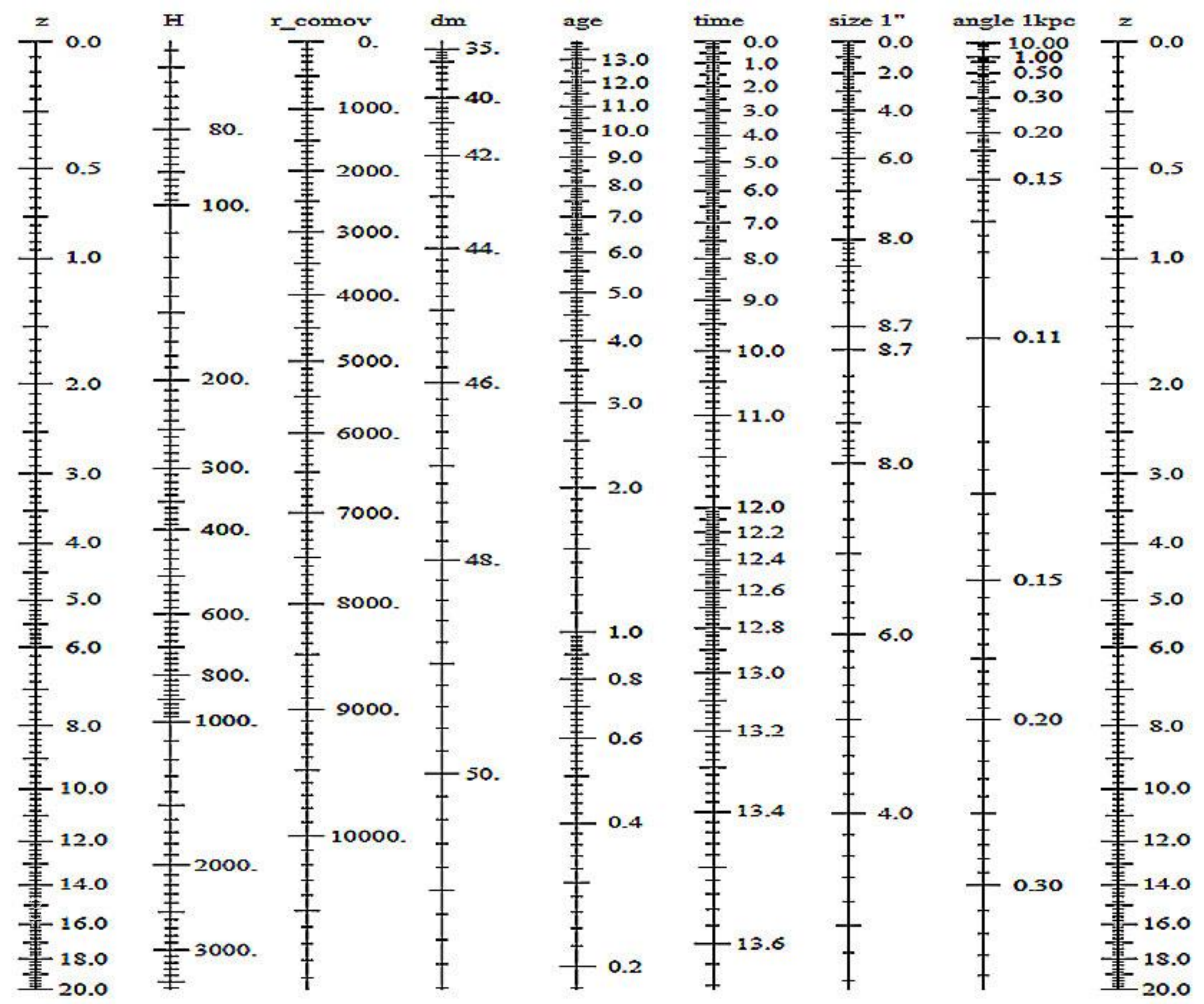


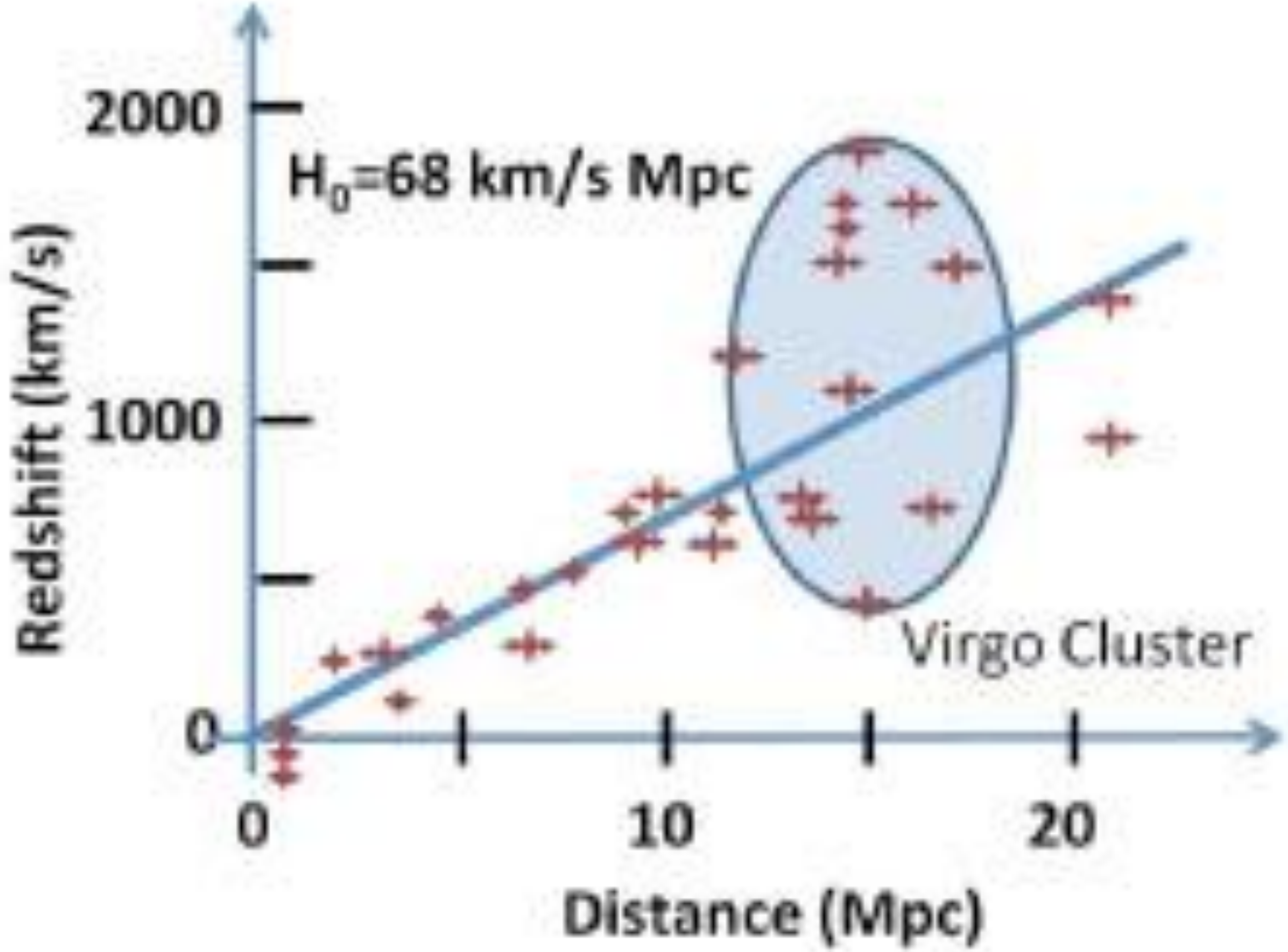


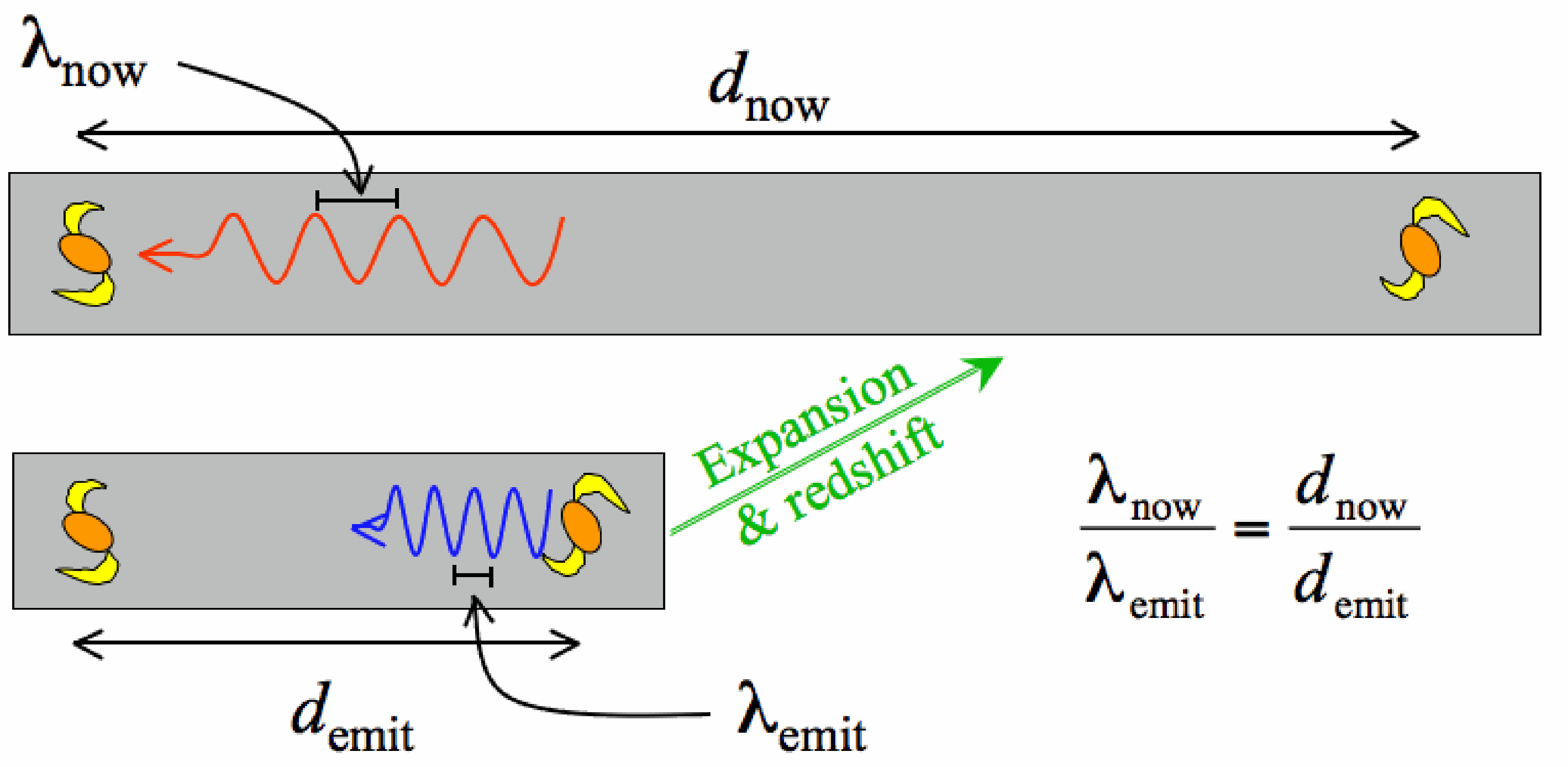


Current Entropy of the Observable Universe (Scheme 1 Entropy Budget)

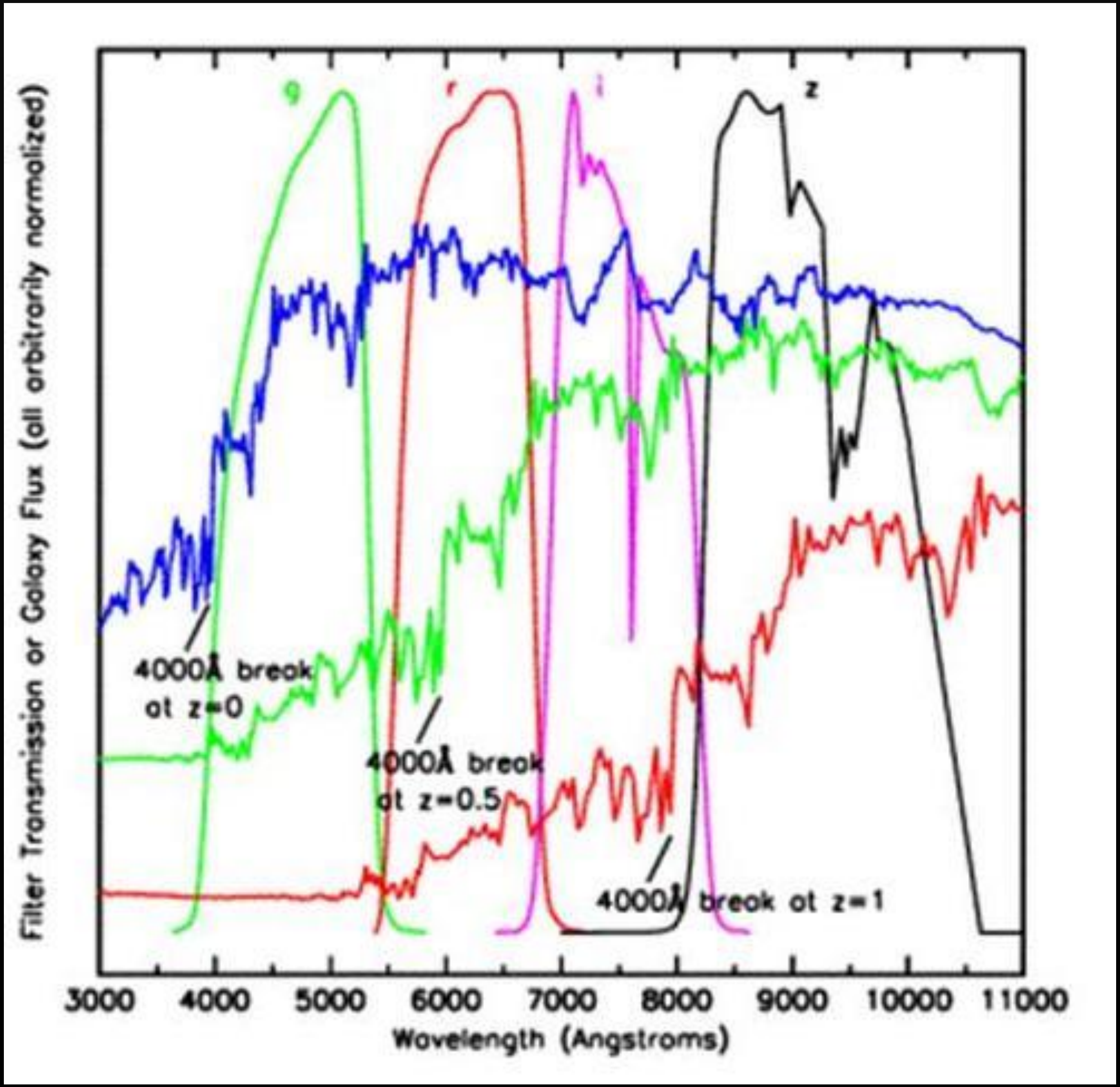
Component	Entropy Density s $[k\ m^{-3}]$	Entropy S $[k]$	Entropy S $[k]$ (previous work)
SMBHs	$8.4^{+8.2}_{-4.7} \times 10^{23}$	$3.1^{+3.0}_{-1.7} \times 10^{104}$	10^{101}[1], 10^{102}[2], 10^{103}[3]
$*$ Stellar BHs ($42-140\ M_{\odot}$)	$8.5 \times 10^{18^{+0.8}_{-1.6}}$	$3.1 \times 10^{99^{+0.8}_{-1.6}}$	–
Stellar BHs ($2.5-15\ M_{\odot}$)	$1.6 \times 10^{17^{+0.6}_{-1.2}}$	$5.9 \times 10^{97^{+0.6}_{-1.2}}$	10^{97}[2], 10^{98}[4]
Photons	$1.478 \pm 0.003 \times 10^{9}$	$5.40 \pm 0.15 \times 10^{89}$	10^{88}[1,2,4], 10^{89}[5]
Relic Neutrinos	$1.411 \pm 0.014 \times 10^{9}$	$5.16 \pm 0.15 \times 10^{89}$	10^{88}[2],10^{89}[5]
Dark Matter	$5 \times 10^{7 \pm 1}$	$2 \times 10^{88 \pm 1}$	–
Relic Gravitons	$1.7 \times 10^{7^{+0.2}_{-2.5}}$	$6.2 \times 10^{87^{+0.2}_{-2.5}}$	10^{86}[2,3]
ISM & IGM	20 ± 15	$7.1 \pm 5.6 \times 10^{81}$	–
Stars	0.26 ± 0.12	$9.5 \pm 4.5 \times 10^{80}$	10^{79}[2]
Total	$\mathbf{8.4^{+8.2}_{-4.7} \times 10^{23}}$	$\mathbf{3.1^{+3.0}_{-1.7} \times 10^{104}}$	**10^{101}[1], 10^{102}[2], 10^{103}[3]**

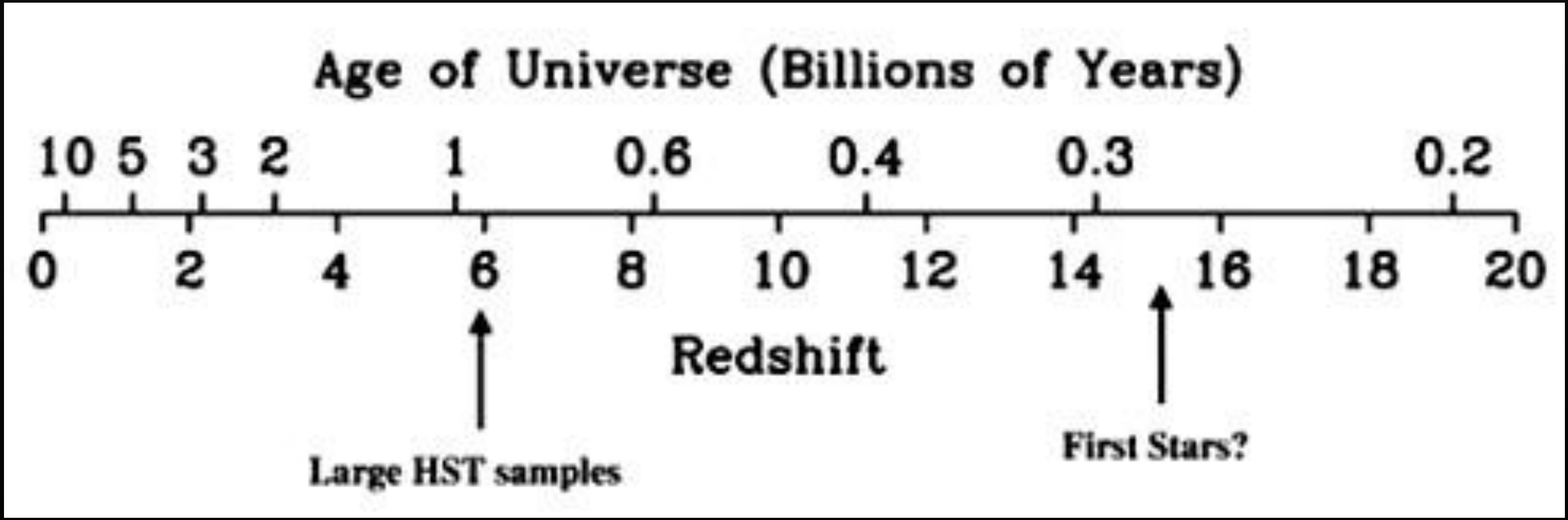

z
0.0
0.5
1.0
2.0
3.0
4.0
5.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
H
80.
100.
200.
300.
400.
600.
800.
1000.
2000.
3000.
r_comov
0.
1000.
2000.
3000.
4000.
5000.
6000.
7000.
8000.
9000.
10000.
dm
35.
40.
42.
44.
46.
48.
50.
age
13.0
12.0
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.8
0.6
0.4
0.2
time
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
12.0
12.2
12.4
12.6
12.8
13.0
13.2
13.4
13.6
size 1"
0.0
2.0
4.0
6.0
8.0
8.7
8.7
8.0
6.0
4.0
angle 1kpc
10.00
1.00
0.50
0.30
0.20
0.15
0.11
0.15
0.20
0.30
z
0.0
0.5
1.0
2.0
3.0
4.0
5.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0

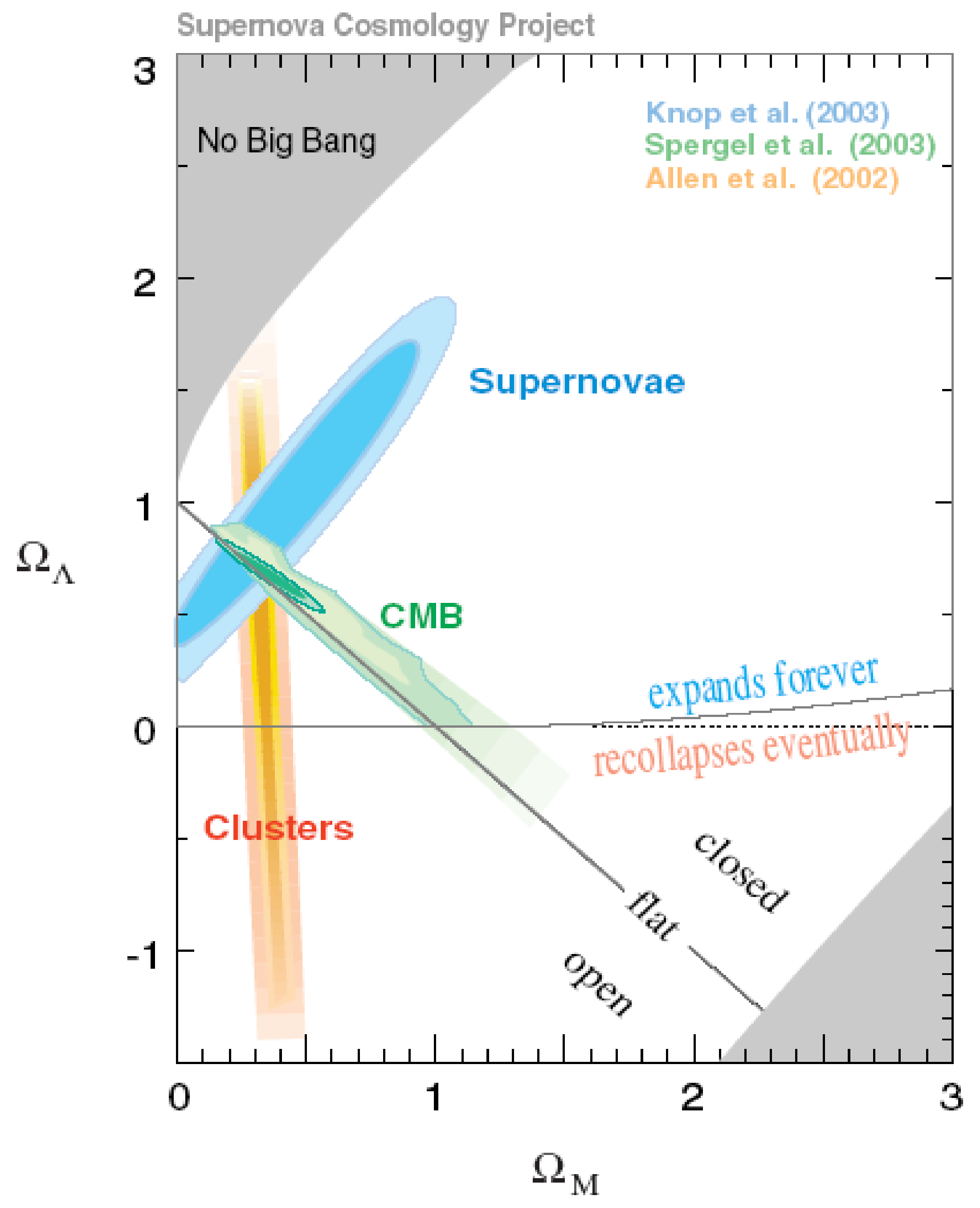

2000
1000
0
Redshift (km/s)
H_0=68 km/s Mpc
Virgo Cluster
0
10
20
Distance (Mpc)



$$\frac{\lambda_{\text{now}}}{\lambda_{\text{emit}}} = \frac{d_{\text{now}}}{d_{\text{emit}}}$$

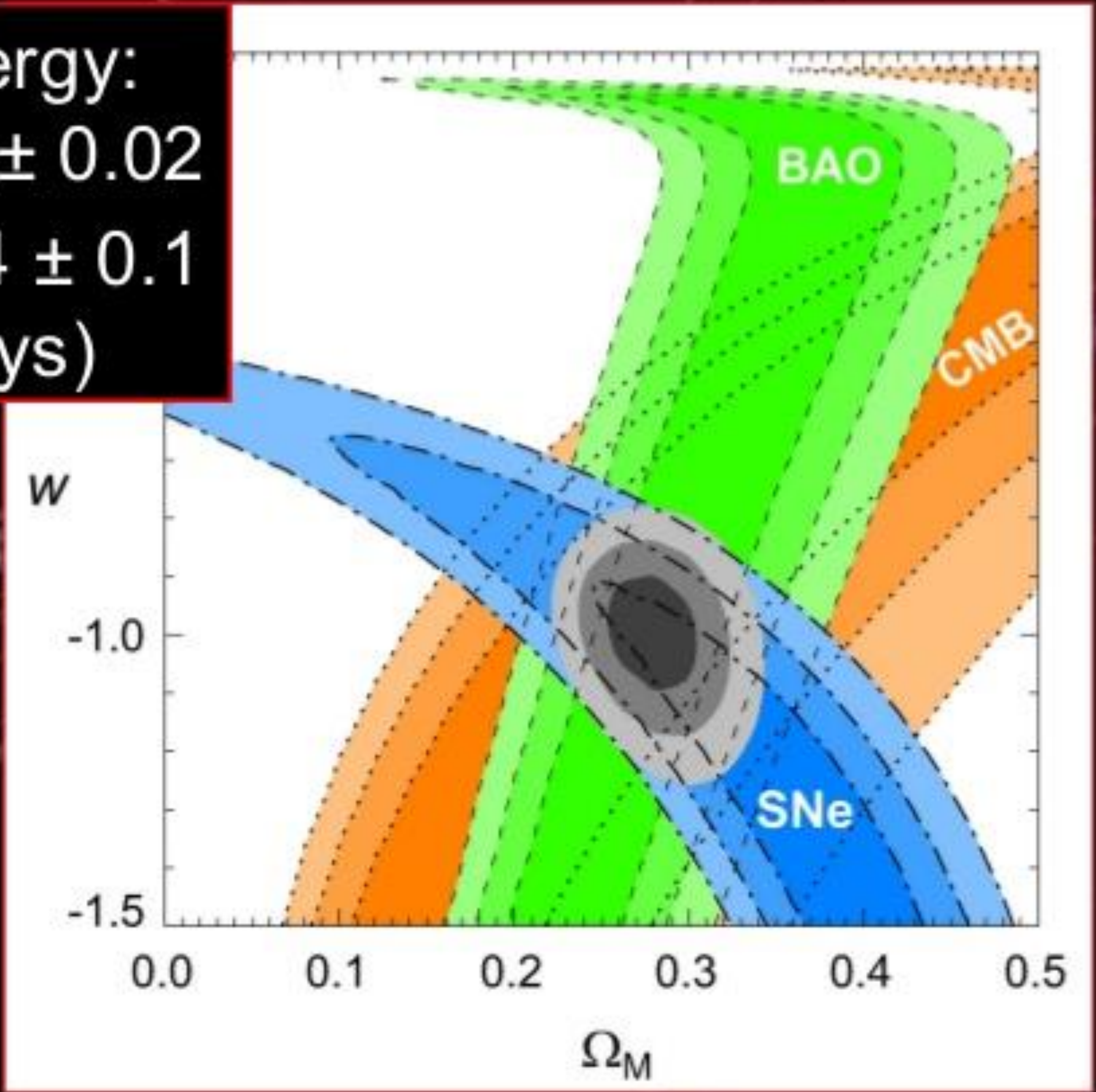

g
r
i
z
4000Å break at z=0
4000Å break at z=0.5
4000Å break at z=1
Filter Transmission or Galaxy Flux (all arbitrarily normalized)
Wavelength (Angstroms)
3000
4000
5000
6000
7000
8000
9000
10000
11000

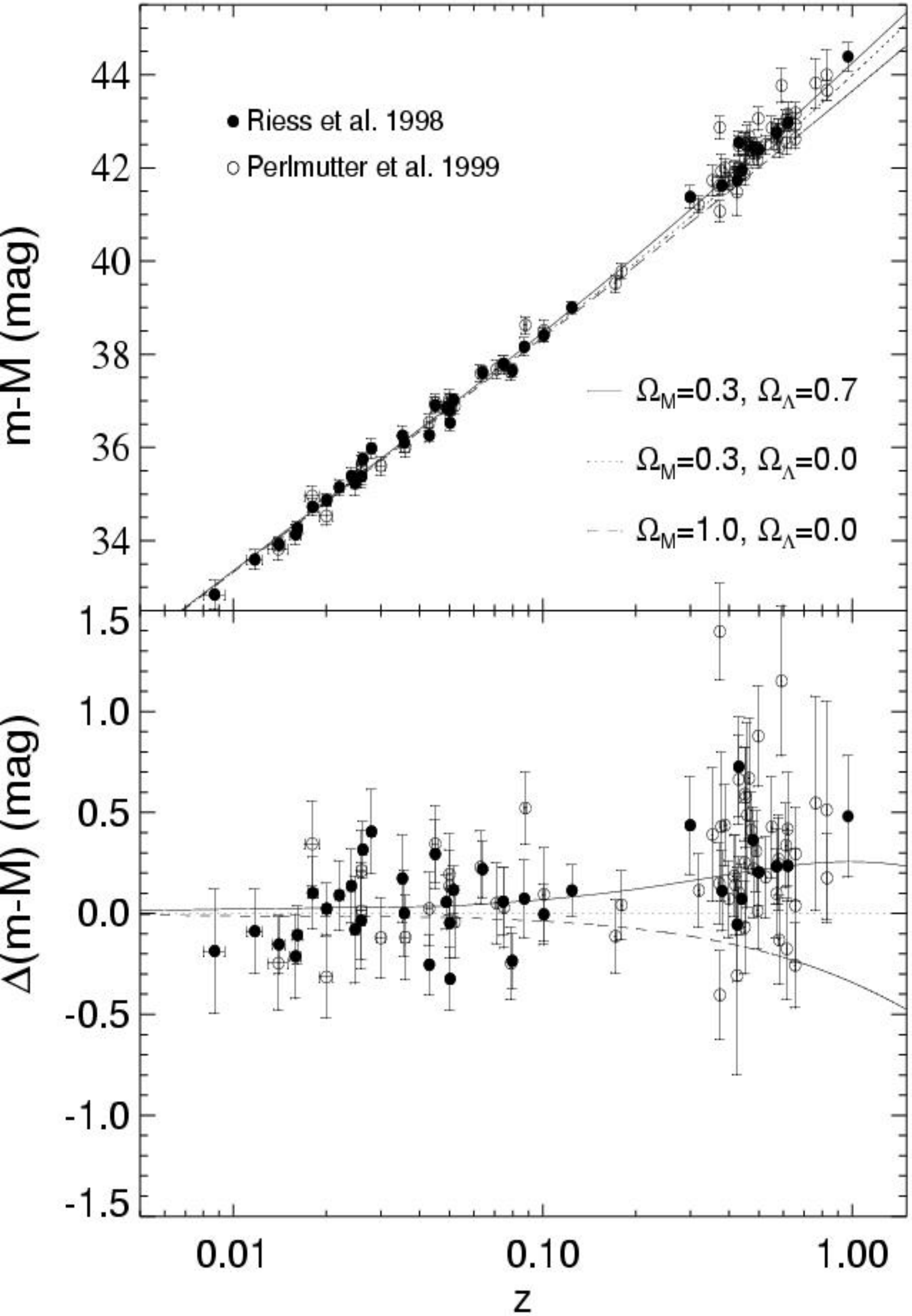

Age of Universe (Billions of Years)
10
5
3
2
1
0.6
0.4
0.3
0.2
0
2
4
6
8
10
12
14
16
18
20
Redshift
Large HST samples
First Stars?

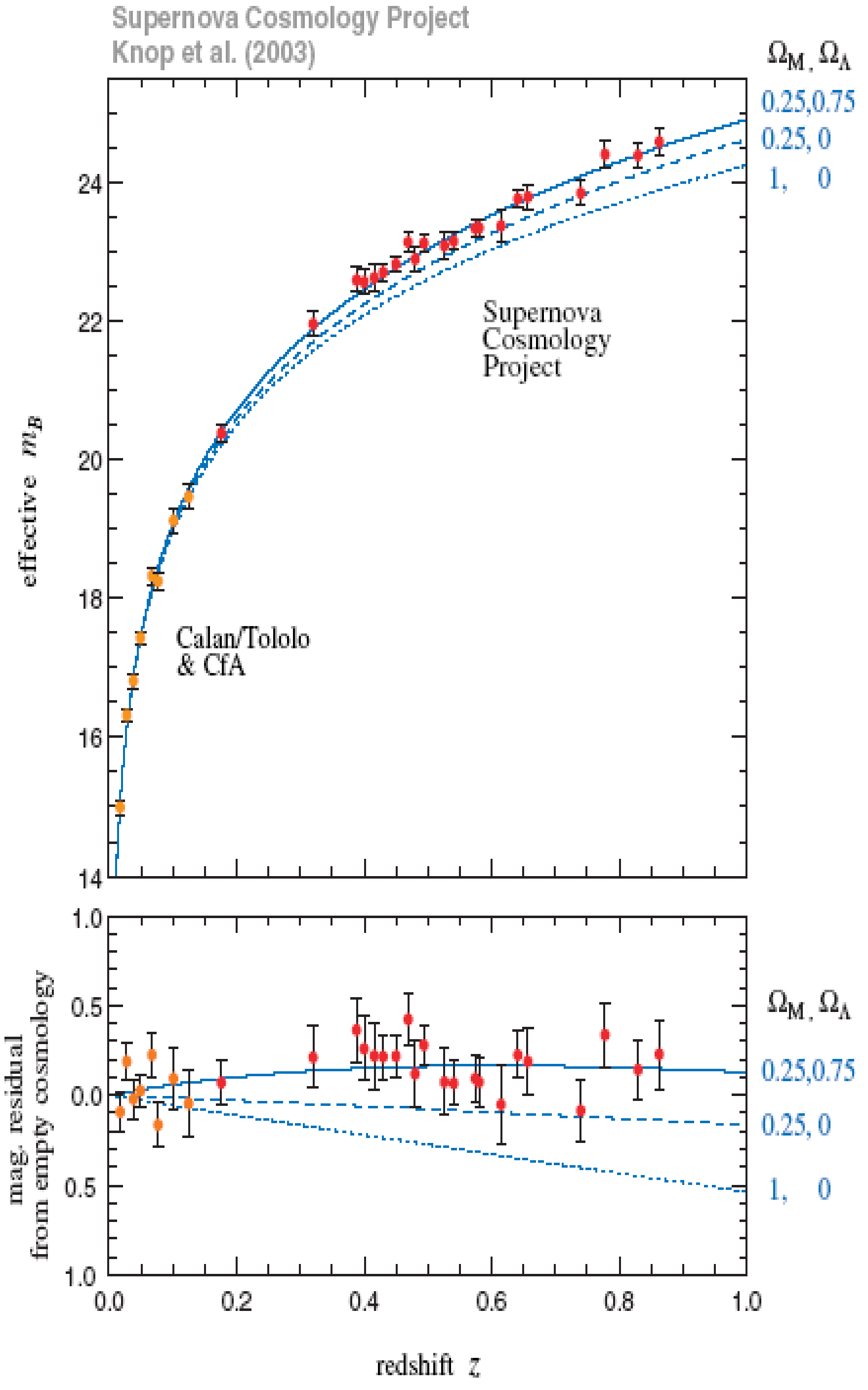

Supernova Cosmology Project
No Big Bang
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
Supernovae
CMB
Clusters
expands forever
recollapses eventually
closed
flat
open
3
2
1
0
-1
0
1
2
3
Ω_Λ
Ω_M

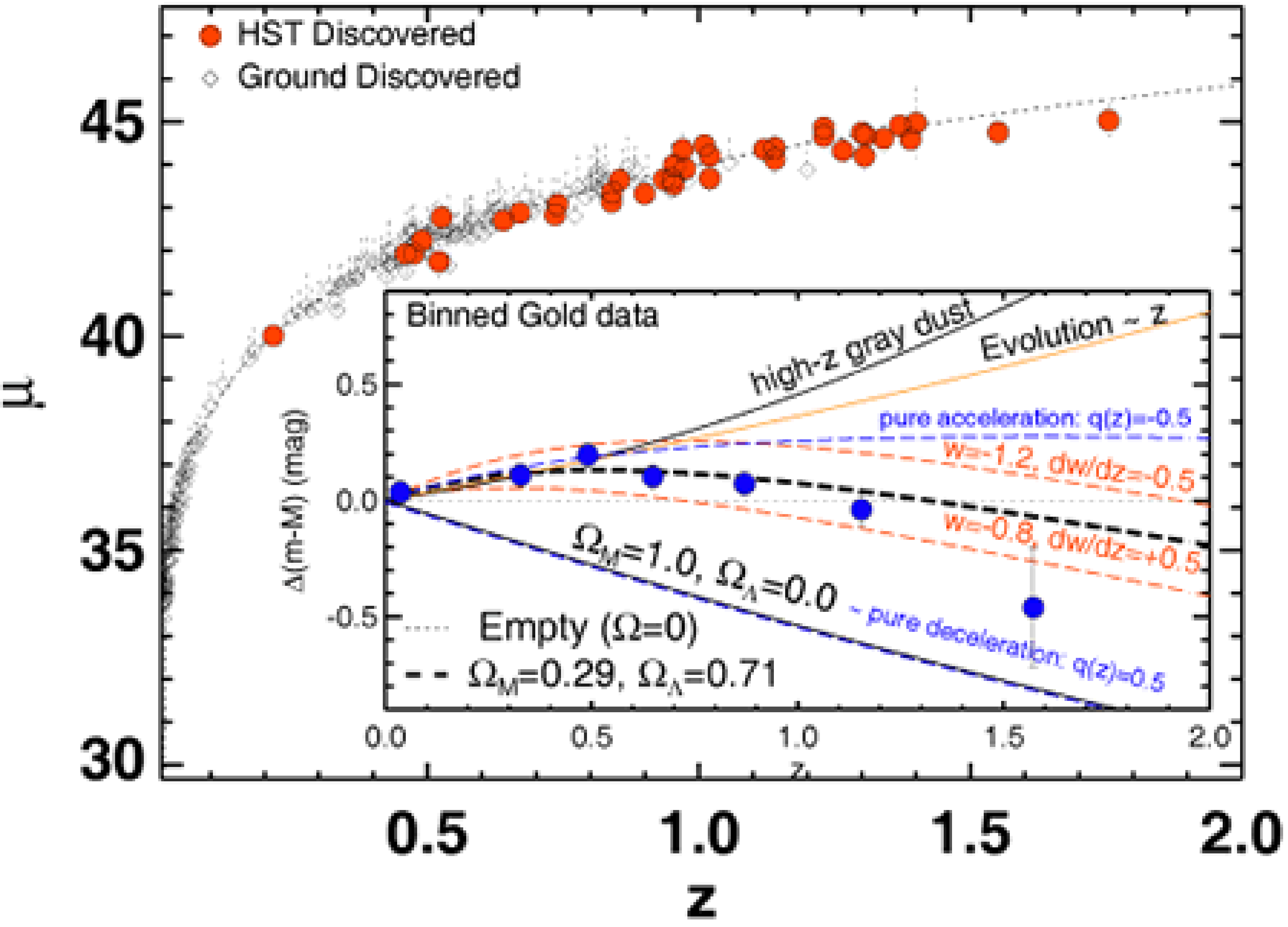
Where We Are Today

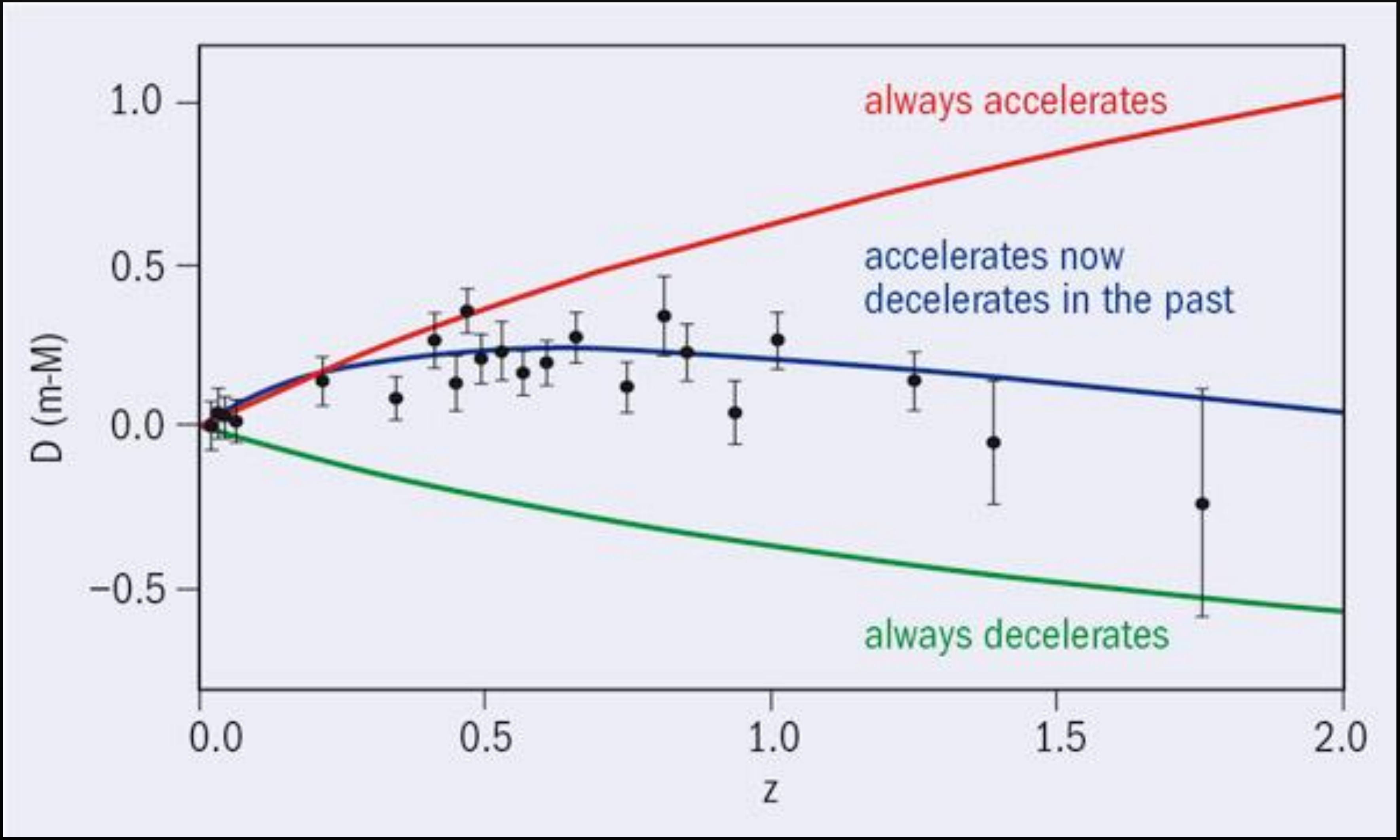
Dark Energy:
$\Omega_{DE} = 0.76 \pm 0.02$
$w = -0.94 \pm 0.1$
(± 0.1 sys)



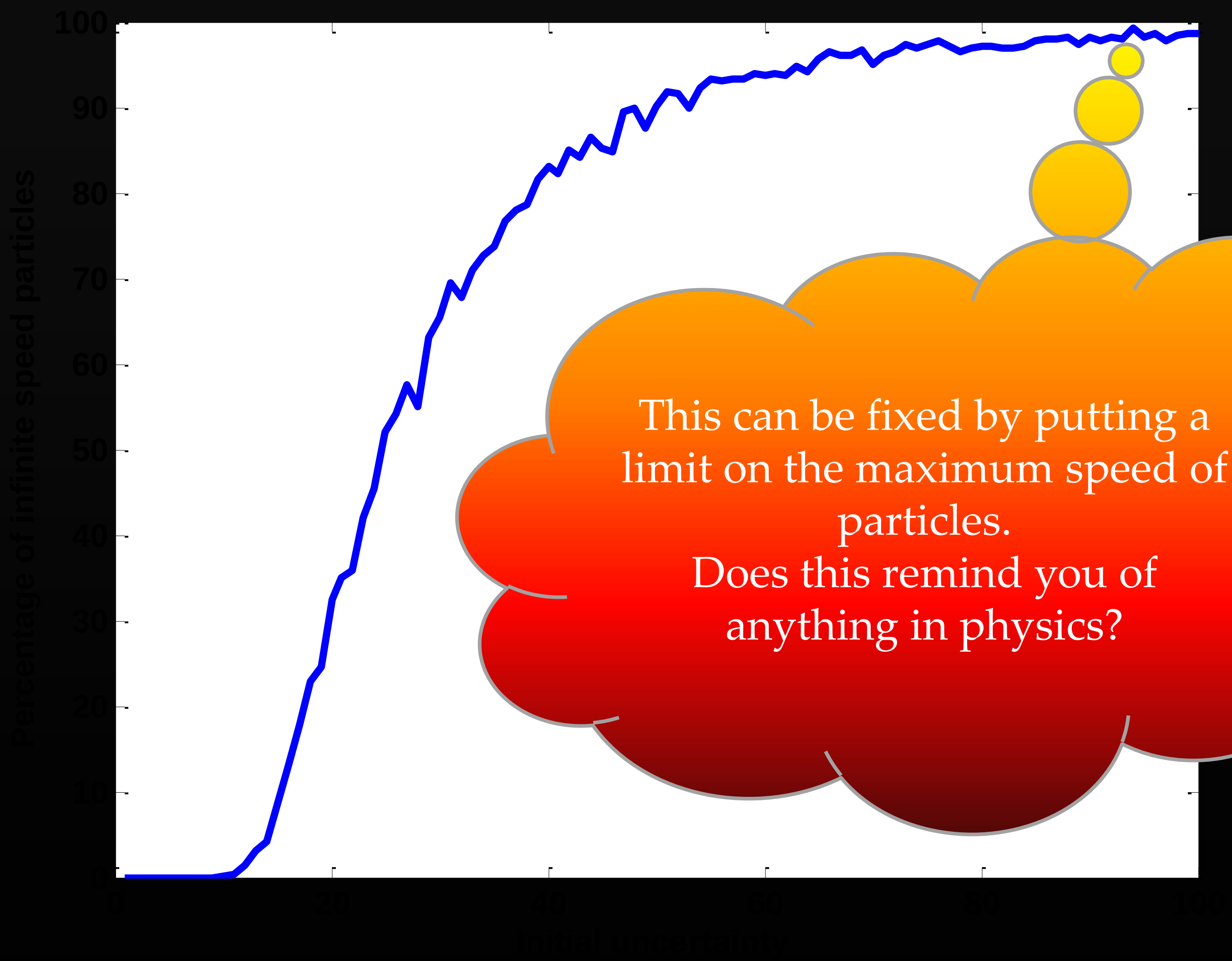



Supernova Cosmology Project
Knop et al. (2003)


HST Discovered
Ground Discovered
μ
45
40
35
30
0.5
1.0
1.5
2.0
z
Binned Gold data
high-z gray dust
Evolution ~ z
pure acceleration: q(z)=-0.5
w=-1.2, dw/dz=-0.5
w=-0.8, dw/dz=+0.5
Ω_M=1.0, Ω_Λ=0.0
~ pure deceleration: q(z)=0.5
Empty (Ω=0)
Ω_M=0.29, Ω_Λ=0.71
Δ(m-M) (mag)
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
z

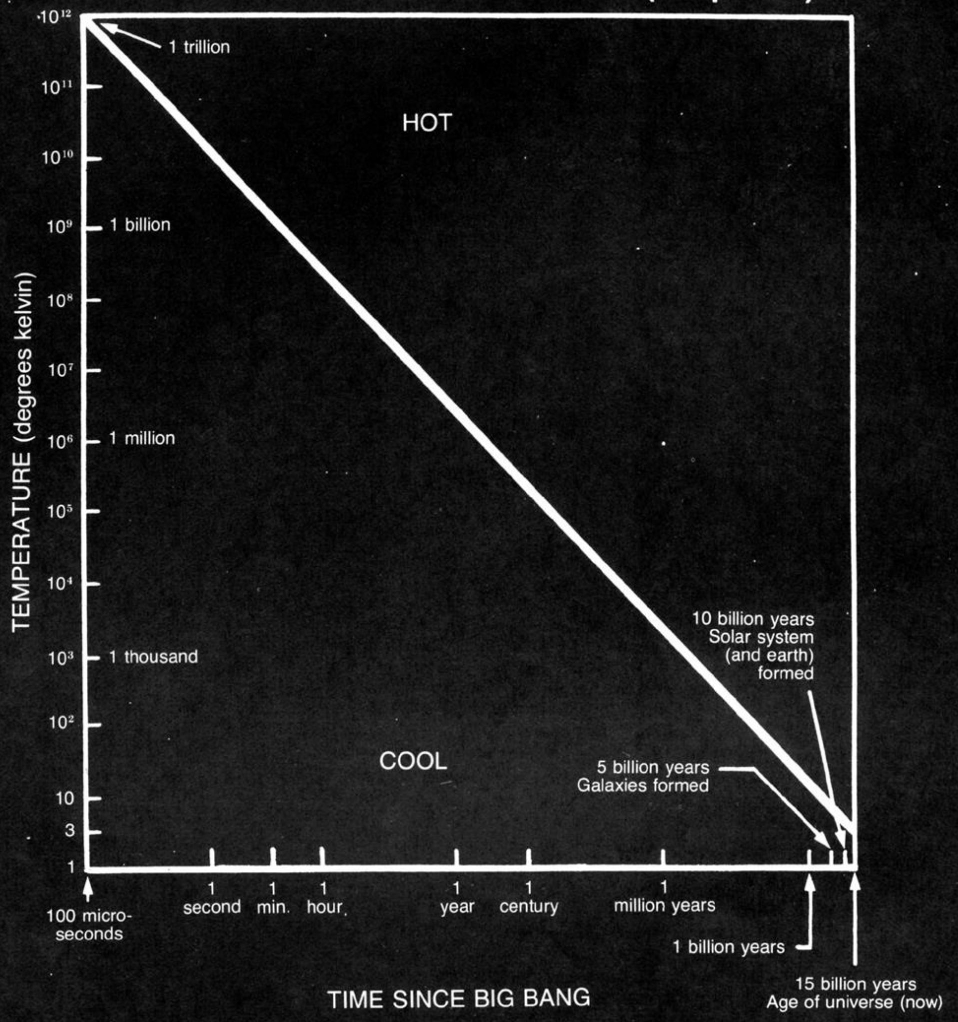


D (m-M)
1.0
0.5
0.0
−0.5
0.0
0.5
1.0
1.5
2.0
z
always accelerates
accelerates now
decelerates in the past
always decelerates



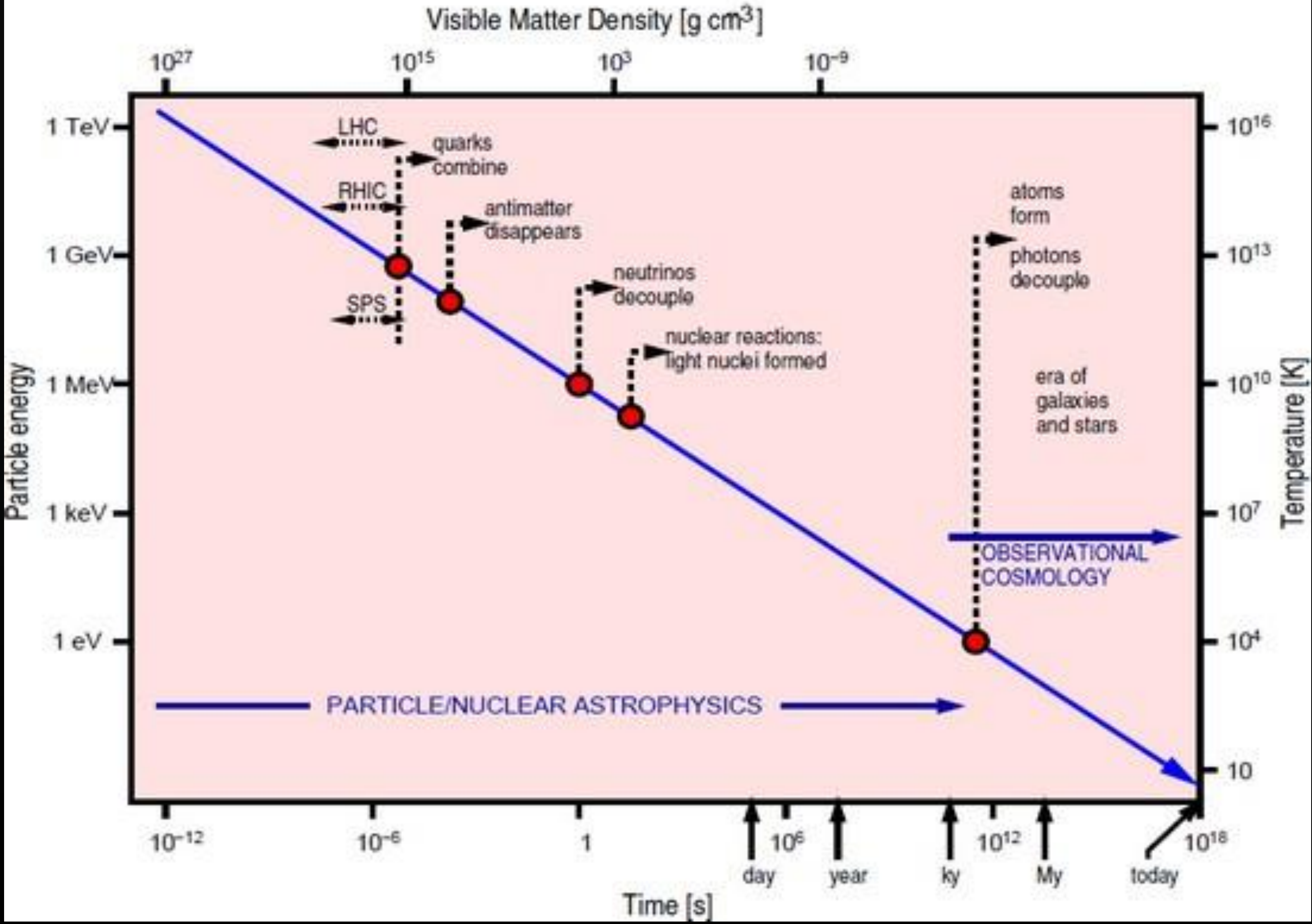
This can be fixed by putting a limit on the maximum speed of particles.
Does this remind you of anything in physics?

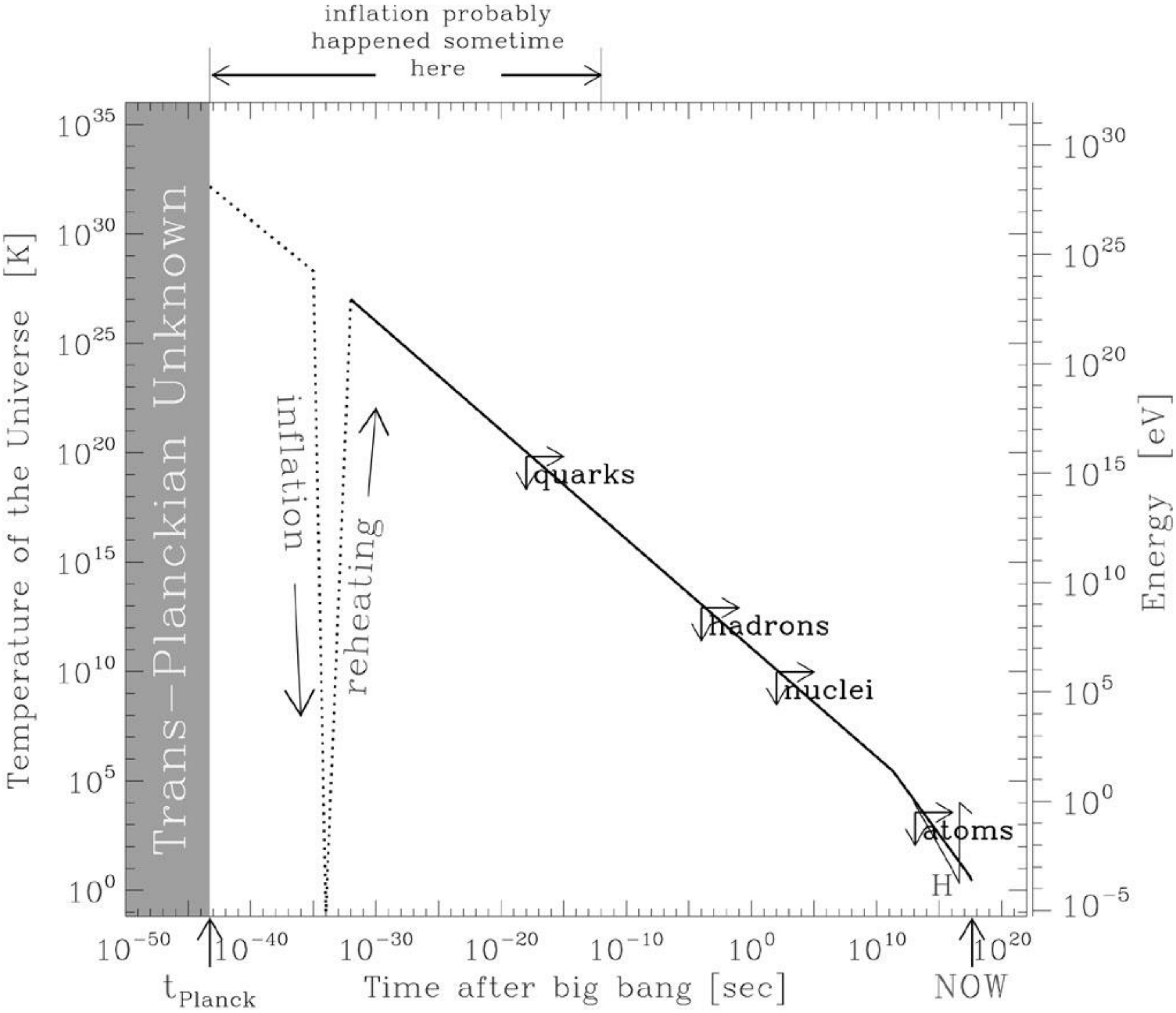
COSMIC TEMPERATURE GRAPH (simplified)

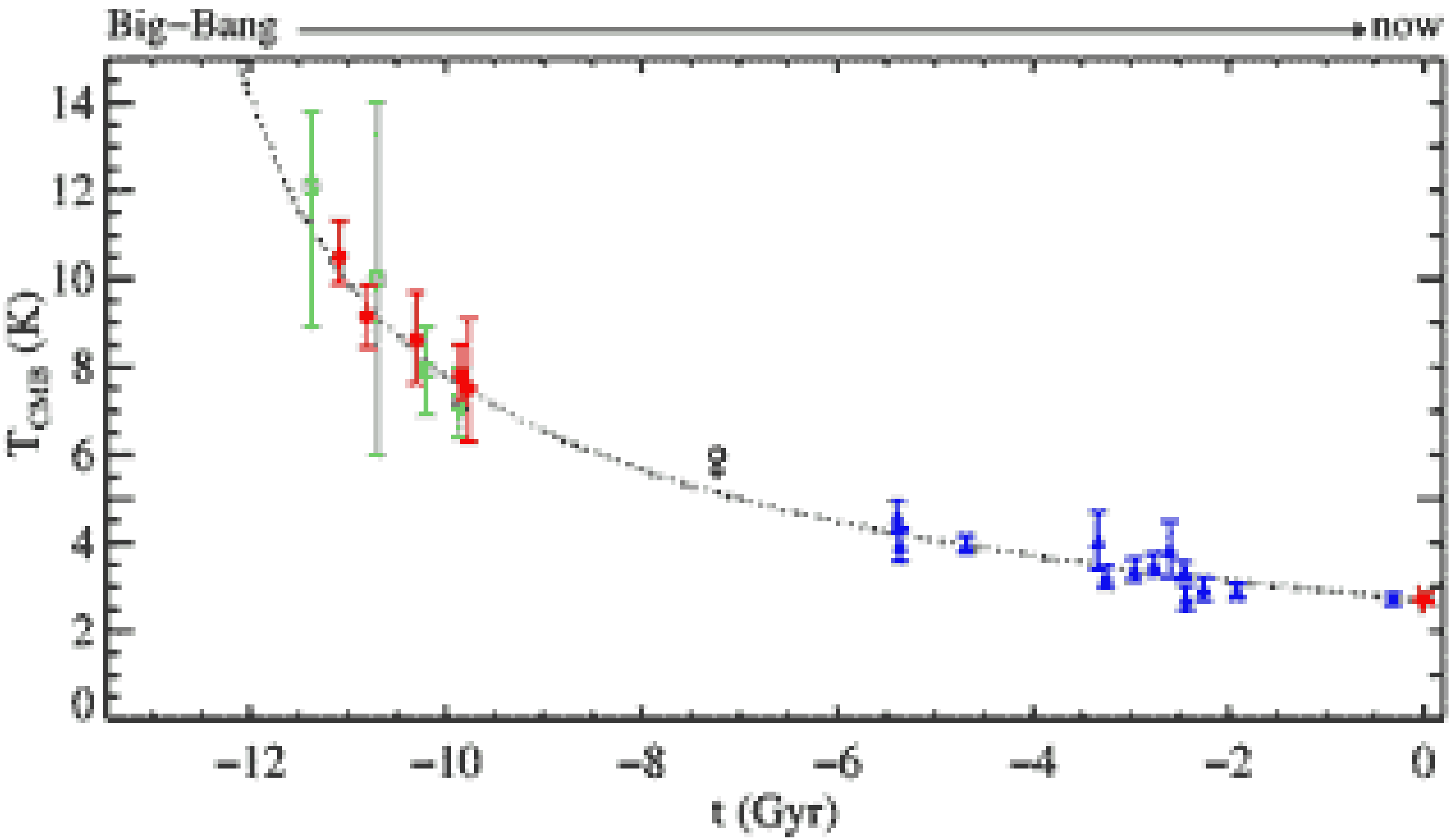


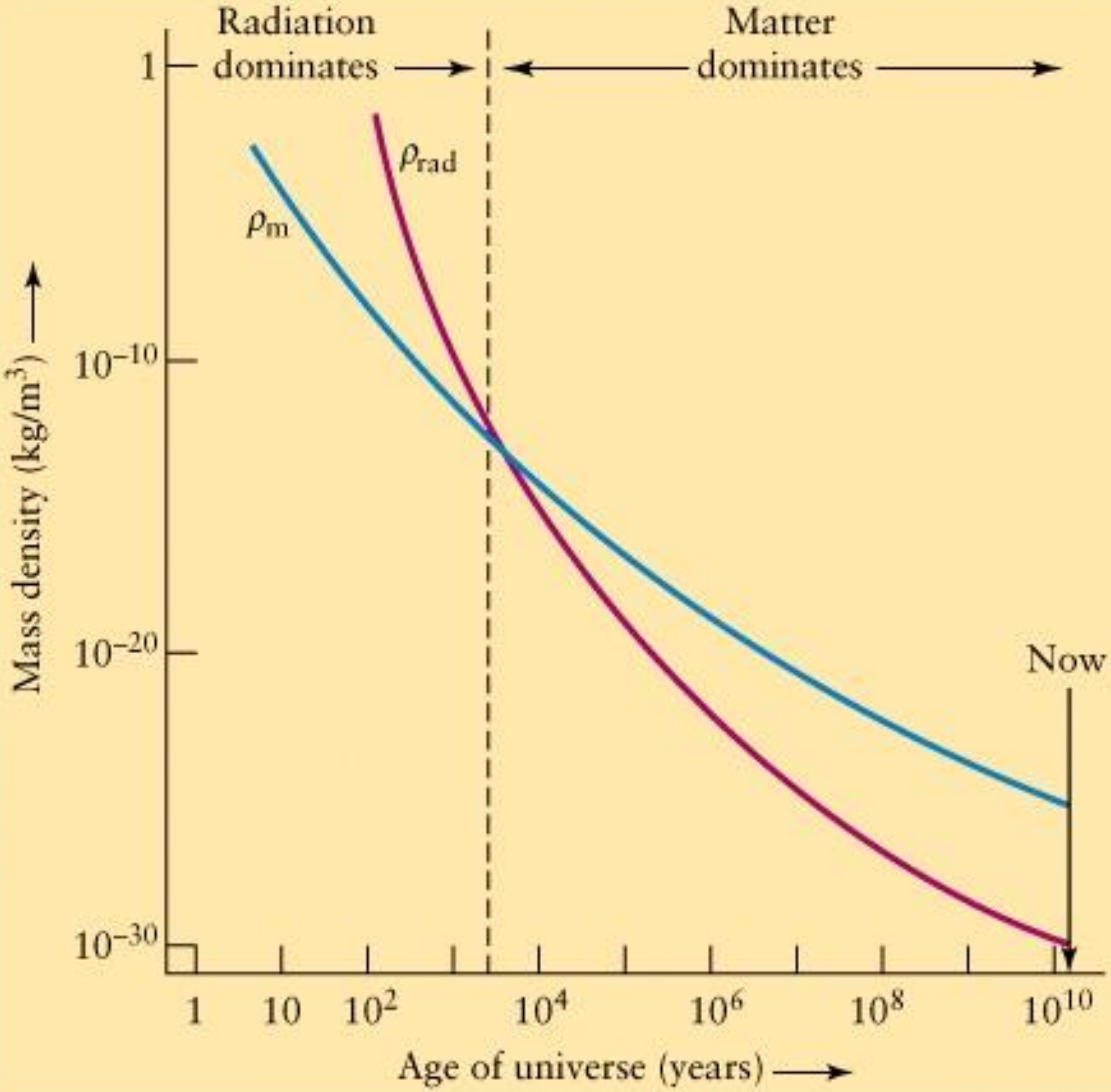
Starting with a temperature of a trillion degrees a few microseconds after the Big Bang (time zero), the universe has cooled down as it expanded until the temperature now is about 3 degrees kelvin (3 degrees celsius above absolute zero). The radius of the universe has now expanded to about 15 billion light-years. (Adapted from R. H. Dicke et al, Astrophysical Journal, vol. 142, page 414, 1965).

Stages in the evolution of the Universe

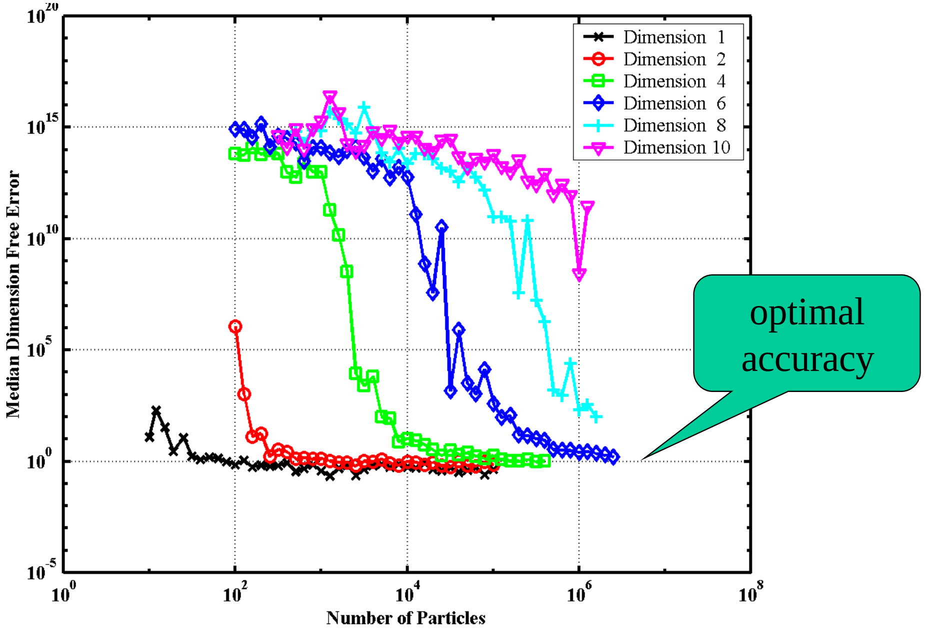



inflation probably happened sometime here
Trans-Planckian Unknown
inflation
reheating
quarks
hadrons
nuclei
atoms
H
Temperature of the Universe [K]
10^{35}
10^{30}
10^{25}
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
Energy [eV]
10^{30}
10^{25}
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
10^{-5}
10^{-50}
10^{-40}
10^{-30}
10^{-20}
10^{-10}
10^{0}
10^{10}
10^{20}
t_{Planck}
Time after big bang [sec]
NOW

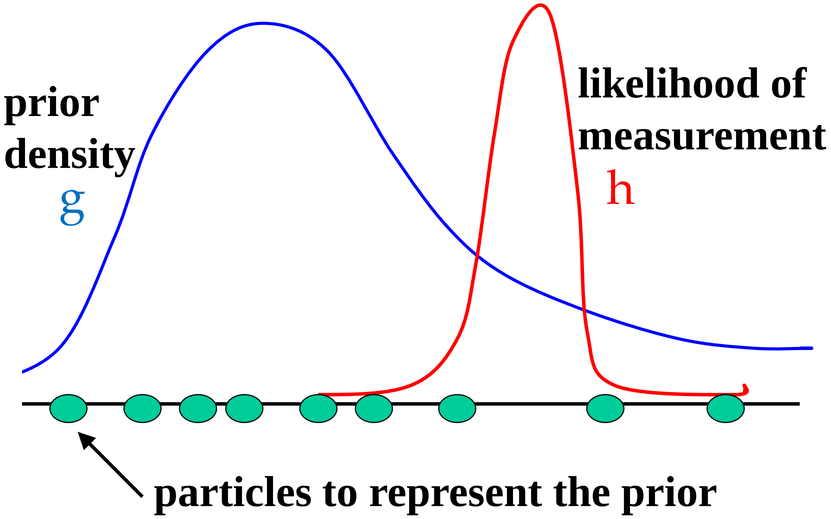

Big−Bang
now
14
12
10
8
6
4
2
0
T_{CMB} (K)
−12
−10
−8
−6
−4
−2
0
t (Gyr)


Radiation dominates
Matter dominates
1
10^{-10}
10^{-20}
10^{-30}
Mass density (kg/m^3)
ρ_{rad}
ρ_m
Now
1
10
10^2
10^4
10^6
10^8
10^{10}
Age of universe (years)

🟩 incompressible flow	🟩 irrotational flow	🟩 Coulomb’s law flow	🟩 small curvature flow	🟦 constant curvature flows (e.g. zero)
🟥 Gaussian densities	🟥 exponential family	🟥 Knothe-Rosenblatt flow	🟥 non-zero diffusion flow	🟦 geodesic flows
🟦 Fourier transform flow	🟦 direct integration	🟦 stabilized flows	🟦 finite dimensional flow	🟦 optimal Monge-Kantorovich transports
🟦 method of characteristics	🟩 renormalization group flow for log $K(\lambda)$ inspired by QFT	🟩 renormalization group flow for log g(x) inspired by QFT	🟩 renormalization group flow for logK(λ) and log g(x)	🟥 exponential family with non-zero diffusion
🟦 Gibbs sampler like flow (inspired by direct integration)	🟦 non-singular Jacobian flow (inspired by proof)	🟦 maximum entropy flow	🟦 Moser coupling flow	🟦 suboptimal Monge-Kantorovich



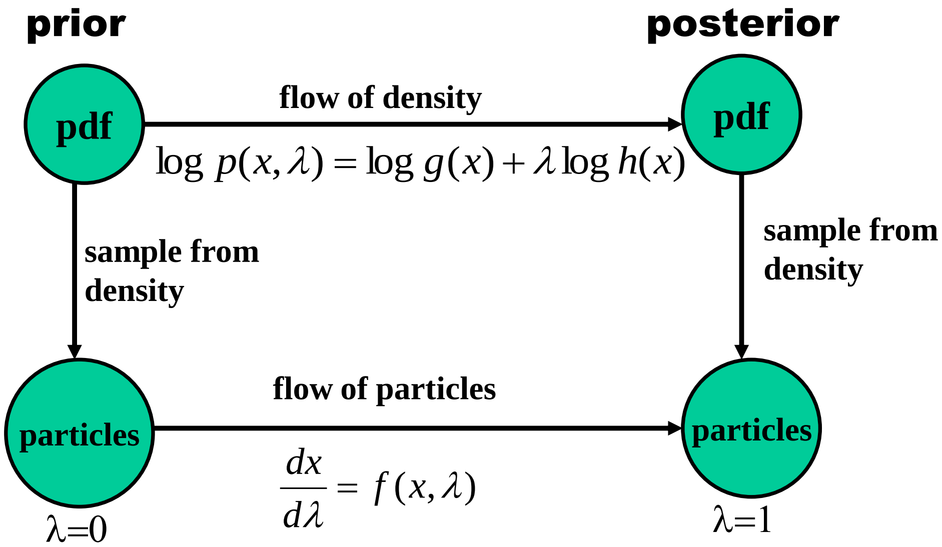
optimal accuracy



prior density
g

likelihood of measurement
h

particles to represent the prior



prior

posterior

pdf

flow of density

$\log\, p(x,\lambda) = \log g(x) + \lambda \log h(x)$

pdf

sample from density

sample from density

particles

flow of particles

particles

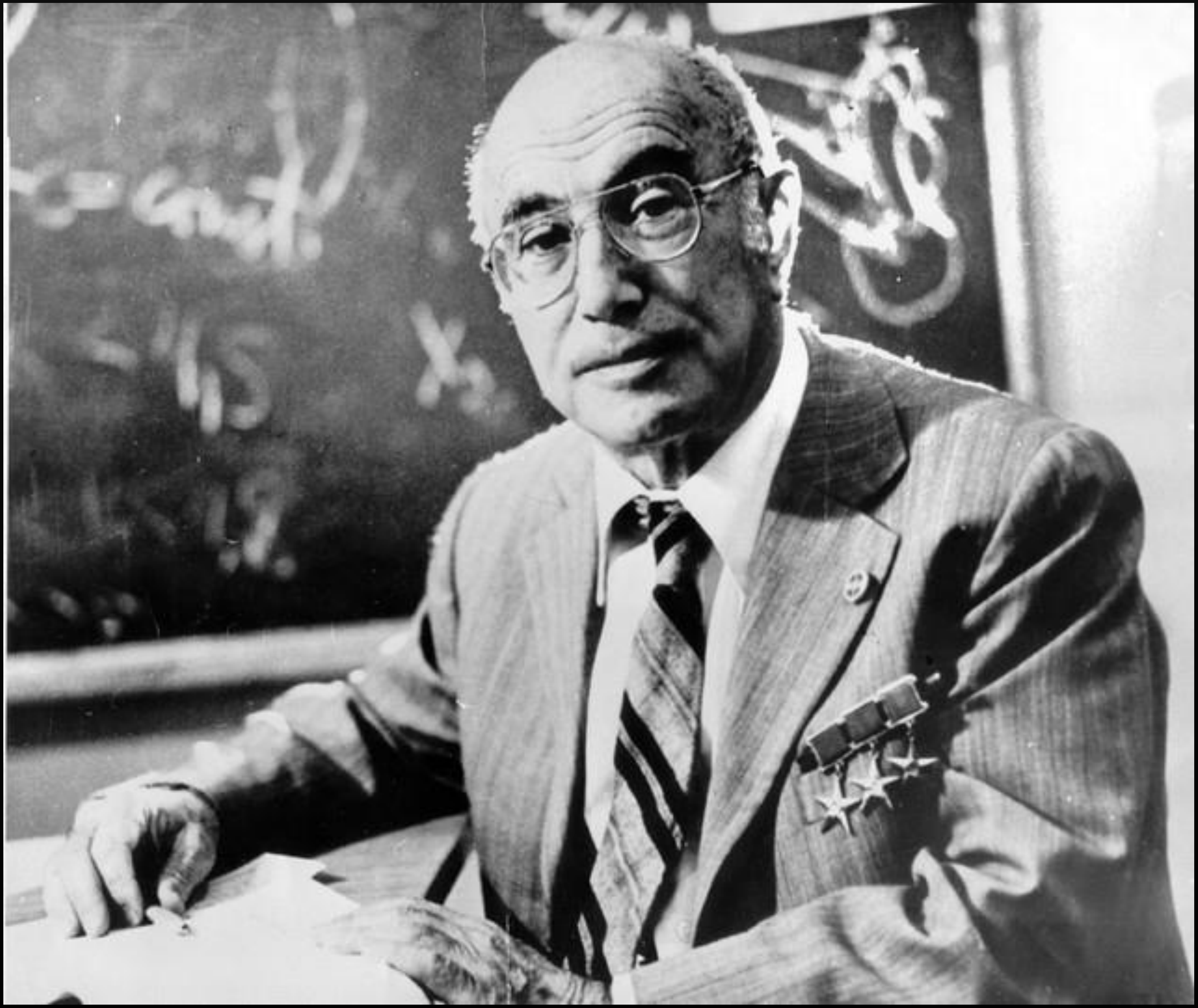
$$\frac{dx}{d\lambda} = f(x,\lambda)$$

λ=0

λ=1

$$div(pf) = p\left[-\log h + \frac{d\log K}{d\lambda}\right]$$

We design the particle flow by solving the above PDE for f.



physicist	math tool	physics
Kepler	ellipse & logarithms	Kepler’s orbital laws
Fermat	geometry & algebra	Fermat’s principle in optics
Newton	calculus & geometry	mechanics
Euler & Lagrange	PDEs	Euler-Lagrange eqs & fluid mechanics
Hamilton	PDEs	Hamiltonian mechanics
Maxwell, Gauss, et al.	PDEs	Maxwell’s eqs
Boltzmann	PDEs	statistical mechanics
Einstein & Hilbert	Riemannian geometry & calculus of variations	general relativity
Schrödinger & Dirac	PDEs	Schrödinger & Dirac equations
Feynman & Dyson	Feynman path integral	QED
Yang & Mills	gauge transformations	Yang-Mills eq
Weinberg, Glashow & Salam	Lie algebras	unification of E & M and weak force
Witten, et al.	super Lie algebras	supersymmetry & string theory
---	transport theory	Bayesian quantum gravity